

Skewness in Stock Returns with Periodic Dividends*

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Abstract

This paper analyzes the asset pricing implications of periodic dividend payments within the context of a stationary rational expectations model with heterogeneous investors. The periodicity of dividends provides a natural motivation for time-varying conditional volatility in stock returns. I show that the unconditional distribution of returns is a mixture of normals distribution, which has non-trivial skewness properties. I examine how conditional volatility, trading volume and skewness in stock returns are related to information dispersion and liquidity in the stock market.

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1 Introduction

This paper develops an asset pricing model under the assumptions of periodic dividend payments and investor heterogeneity. The periodicity of dividend payments motivates conditional heteroskedasticity in stock returns. I use the model to study how investor heterogeneity affects conditional heteroskedasticity, trading volume and skewness in stock returns. I am interested in analyzing the sources of skewness in stock returns, and the relation between skewness and trading volume.

Periodicity in dividend payments generates positive skewness in stock returns. With periodic dividends, cash flow news is priced to reflect the appropriate discounting up to the dividend payment.¹ Thus, news released closer to the dividend payment produces a greater impact on the conditional return volatility, even though the unconditional volatility of news is constant. Not only the conditional volatility of returns increases towards the dividend payment, but also it increases at an increasing rate. In the presence of a risk-return trade off in the model, these properties apply also to expected returns and induce positive skewness in expected returns.

In the model, investors are heterogeneous with respect to their information sets and their investment opportunity sets. Building on Wang (1994), informed investors are allowed to trade a stock and a private investment opportunity. The private investment return is positively correlated with the stock return giving rise to rebalancing trades. These rebalancing trades mask the information trades by the same informed investors and prevent the equilibrium from being fully revealing.

Shocks to the private investment return mimic liquidity shocks in that they give rise to non-informative, rebalancing trades that need to be absorbed by the remaining investors in the market. The incentive to rebalance the portfolio changes through time and is driven by the time series pattern of the conditional correlation of stock returns with the private investment return. When liquidity shocks are large, this correlation inherits the monotonicity in the conditional covariance between the stock return and the private investment. In particular, the conditional covariance increases with time up to the dividend payment because with less discounting the stock can serve as a better hedge to the private investment. With an increasing conditional correlation, informed investors optimally choose to hold more of the stock. The equilibrium then requires that informed investors bid up the stock price well

¹In the model announcement of dividends coincides with payment of dividends.

before the dividend payment, offsetting the steep increase in expected returns that otherwise would exist due to the periodicity in dividends.

The presence of asymmetric information gives informed investors an added incentive to buy more of the stock as the date of the dividend payment nears. This is because the information asymmetry is highest just before the dividend payment, despite some information revelation through the market price. Again, this means that at the equilibrium informed investors offer high prices long before the dividend announcement. Both liquidity shocks and information asymmetry thus work to flatten expected returns closer to dividend payments, reducing skewness in expected returns.

I show that skewness in expected returns is an important determinant of skewness in stock returns. At any point in time, stock returns are conditionally normally distributed with conditional moments that depend, among other things, on the time left to the next dividend payment. Unconditionally, however, stock returns are not normal because the mean and variance of a randomly drawn stock return observation depend on the time left to the next dividend. Instead, the unconditional distribution of stock returns is shown to be a mixture of normals distribution. Under a mixture of normals distribution, skewness in stock returns is given by two components. One is skewness in expected returns. The other captures the association between expected returns and conditional return variance. Numerical exercises of comparative statics suggest that skewness in expected returns can be an important determinant of skewness in stock returns. Therefore, while the assumption of periodic dividends tends to generate positive skewness in stock returns, liquidity shocks and information asymmetry tend to reduce this skewness.

In the model, it is not necessary that greater liquidity shocks lead to greater turnover, because liquidity shocks occur from rebalancing trades. In particular, when liquidity shocks are sufficiently large, the correlation of stock returns with the private investment return is so low –a variance effect– that the incentive to rebalance is reduced and turnover decreases. But for low levels of the liquidity shock, the usual result applies that larger liquidity shocks lead to more turnover. In addition, turnover increases also because larger liquidity shocks reduce uninformed investors' adverse selection problem. Thus, the model has the ability to predict that skewness and turnover are negatively associated when this association is driven by liquidity shocks.

Turning now to existing empirical evidence, it is well documented that firm-level, daily stock returns display positive skewness (e.g., Kon, 1984, Chen et al., 2001, and Bris et al.,

2007), though marketwide returns display negative skewness. In addition, recent studies have shown that skewness in firm-level returns is negatively associated with own firm turnover (Chen et al., 2001). There is also evidence that firm level stock returns are well described by a mixture of normals distribution (Kon, 1984, Zangari, 1996, and Haas et al., 2004).

The theoretical literature has focused on explaining instances of negative skewness on marketwide returns, for example linked to market crashes. Hong and Stein (2003) argue that negative skewness in stock returns arises because the presence of short sales constraints limits the ability of bad news to enter the market, thus keeping prices artificially too high. That is, until the bad news becomes prevalent and prices crash to reflect the cumulative information.² Hong and Stein go on to argue that this effect is not the sole determinant of skewness in their model but that it dominates when differences of opinion are large. They thus predict that skewness and turnover should be negatively related.

Recently, however, Bris et al. (2007) do not reject the hypothesis that firm-level skewness is unrelated to the presence of short sales restrictions (likewise, see Bae et al., 2006). Together with the fact that firm-level skewness is positive, the firm-level evidence calls for a theory that explains the association between skewness and trading volume. This paper provides one such theory. In addition, I also argue that the negative association between skewness in stock returns and trading volume can occur even at low levels of information asymmetry, further distinguishing from the mechanism in Hong and Stein.

The model is also consistent with other facts on dividend announcements and earnings announcements. Dividend announcements are associated with high returns and high volatility of stock returns (Kalay and Loewenstein, 1985). Ball and Khotari (1991) show that the high expected returns around earnings announcements are also associated with high volatility. Finally, informed investors tend to buy before earnings announcements and to sell afterwards. Kaniel et al. (2008) show that individuals intense buying prior to earnings announcements is associated with positive abnormal returns following the announcement and post-announcement sales by individuals. They attribute about 50% of their finding to private information trading.

The model is related to the literature that analyzes the flow of information and its implications for trading behavior (e.g., He and Wang, 1995), and the literature that studies trading

²Blanchard and Watson (1982) suggest that negative skewness is caused by the large but infrequent negative returns that arise when stock price bubbles burst. Other studies that focused on asymmetric volatility include: Black (1976) and Christie (1982) with the leverage effect, Pindyck (1984), French et al. (1987) and Campbell and Hentschel (1992) with the volatility feedback effect, and Bekaert and Wu (2000) and Wu (2001).

volume around public news events in models with information asymmetries (e.g., Kim and Verrecchia, 1991, 1994, 1997, and Kandel and Pearson, 1995). This paper complements that literature by studying a stationary asset pricing model of event studies and focusing on different a set of moments on returns and trading. Albuquerque and Miao (2009) study a framework where investors also have private information about future dividends that is uncorrelated with current dividends. They use that setting to study momentum and reversals in stock returns.

The paper is organized as follows. The next section describes the complete model. Section 3 describes the equilibrium in the complete model. Section 4 analyzes the effect of periodic dividends, liquidity and information asymmetry on conditional moments of stock returns and trading volume. Section 5 discusses the sources of skewness in the model and its association with trading volume. Finally, Section 6 concludes. The appendix contains the proofs of the propositions in the main text and some additional results.

2 The Model

The model economy is composed of infinitely lived investors which have differential information about the underlying value of the stock. Investors also differ in their access to investment opportunities à la Wang (1994). The model is further detailed next.

2.1 Investment opportunities

Investors trade in a riskless asset which has a perfectly elastic supply at the gross rate of return $R > 1$. There is a single stock in fixed supply of 1. Each share of the stock is infinitely divisible and trades competitively in the stock market at the ex-dividend price P_t , at time t . The stock pays a dividend every $K + 1$ periods,

$$D_t = F_t + \sum_{j=0}^K \varepsilon_{t-K+j}^D.$$

It is understood that if t corresponds to a period $k > 0$, then $D_t = 0$. The dividend can be decomposed into a persistent component,

$$F_t = \rho_F F_{t-1} + \varepsilon_t^F, \quad 0 \leq \rho_F \leq 1,$$

with $\varepsilon_t^F \sim N(0, \sigma_F^2)$, and a transitory component, $\sum_{j=0}^K \varepsilon_{t-K+j}^D$. Information about the transitory component is revealed every period in the form of shocks $\varepsilon_t^D \sim N(0, \sigma_D^2)$. Trading

periods are further identified by a superscript $k = 0, \dots, K$, where $k = 0$ refers to a dividend-paying period and $k = 1, \dots, K$ refer to non-dividend-paying periods. Below, I often identify a period with (t, k) . I then write the excess return in a dividend-paying period as

$$Q_t^0 \equiv P_t^0 + D_t - RP_{t-1}^K,$$

and in a non-dividend-paying period as

$$Q_t^k \equiv P_t^k - RP_{t-1}^{k-1}.$$

Every period, investors get two public signals about the next dividend,

$$\begin{aligned} S_t^{Fk} &= F_t + \varepsilon_t^{Fk}, \\ S_t^{Dk} &= \varepsilon_t^D + \varepsilon_t^{Dk}, \end{aligned}$$

where $\varepsilon_t^{Fk} \sim N(0, \sigma_{Fk}^2)$ and $\varepsilon_t^{Dk} \sim N(0, \sigma_{Dk}^2)$. The quality of the public information is determines the level of asymmetric information. When information is infinitely precise and $\sigma_{Dk}^2 = \sigma_{Fk}^2 = 0$, then there is no asymmetric information across investors.

Lastly, there is another risky asset only available to informed investors which pays the excess return,

$$q_{t+1} = Z_t + \varepsilon_{t+1}^q, \tag{1}$$

at time $t + 1$. The excess return in this private investment opportunity is composed of a persistent expected excess return,

$$Z_t = \rho_Z Z_{t-1} + \varepsilon_t^Z, \quad 0 \leq \rho_Z \leq 1,$$

with $\varepsilon_t^Z \sim N(0, \sigma_Z^2)$ and a transitory unexpected return $\varepsilon_t^q \sim N(0, \sigma_q^2)$. It is assumed that $E(\varepsilon_t^D \varepsilon_t^q) = \sigma_{Dq} > 0$. Except for the correlation between ε_t^D and ε_t^q no other correlation between shocks exists.

2.2 Investors' problem

There is a continuum of identical, informed investors denoted by the superscript i , and a continuum of identical, uninformed investors denoted by the superscript u . The mass of informed investors is λ and the mass of uninformed investors is $1 - \lambda$. Investors choose their time t asset allocation to maximize utility over next period wealth,

$$-E \left[\exp^{-\gamma W_{t+1}} | \mathcal{I}_t^j \right].$$

For informed investors, $j = i$, the maximization is over the holdings of the riskless asset, of the stock, θ_t , and of the private investment opportunity, α_t , and is subject to the budget constraint,

$$W_{t+1} = Q_{t+1}^{k+1} \theta_t + \alpha_t q_{t+1} + RW_t,$$

and the information set,

$$\mathcal{I}_t^i = \{P_{t-s}, D_{t-s}, S_{t-s}^F, S_{t-s}^D, F_{t-s}, Z_{t-s}, \varepsilon_{t-s}^D\}_{s \geq 0}.$$

Uninformed investors, $j = u$, face a similar budget constraint, but with $\alpha_t \equiv 0$, and have the information set,

$$\mathcal{I}_t^u = \{P_{t-s}, D_{t-s}, S_{t-s}^F, S_{t-s}^D\}_{s \geq 0}.$$

For any variable x_t , its conditional expectation under \mathcal{I}_t^u is denoted by $\hat{x}_t = E^u[x_t | \mathcal{I}_t^u] = E_t^u[x_t]$.

2.3 Definition of equilibrium

Investors trade the stock competitively in the stock market, making their asset allocation while taking prices as given. In equilibrium, the stock price is such that the market for the stock clears:

$$\lambda \theta_t^i + (1 - \lambda) \theta_t^u = 1. \quad (2)$$

3 Stock Market Equilibrium

3.1 The equilibrium state vector and price function

In solving for the stock market equilibrium, I start with a guess for the state vector; the vector that contains the information needed to price the stock. In principle, the full history of shocks may be relevant, but because of the information hierarchy present in the information sets of the two investor groups, the equilibrium may be described via a finite-dimensional state vector. To further understand the components of the state vector, consider for example the information relevant at time $(t, 0)$, a dividend-paying period. Because dividends have just been paid, the information contained in $\sum_{j=0}^K \varepsilon_{t-K+j}^D$ is not useful to forecast the next dividend payment and information about F_t and Z_t suffices. Next, consider the period immediately after a dividend-paying period. Because investors now have information about F_t , Z_t , and ε_t^D , and ε_t^D is to be paid later, then all three are needed. In keeping with this

reasoning, I guess the state vector to be

$$\mathbf{x}_t = [F_t, Z_t, \varepsilon_t^D, \dots, \varepsilon_{t-K}^D]^\top.$$

The state vector has a fixed dimension of $K+3$. As suggested above, at times this state vector may contain too much information, but this formulation guarantees stationarity. Letting $\varepsilon_t^k = [\varepsilon_t^F, \varepsilon_t^Z, \varepsilon_t^D, \varepsilon_t^q, \varepsilon_t^{Fk}, \varepsilon_t^{Dk}]^\top$, with $\Sigma_{\varepsilon\varepsilon}^k = E[\varepsilon_t^k \varepsilon_t^{k\top}]$, the appendix shows that the dynamics of the state vector can be represented via the constant matrices \mathbf{A}_x and \mathbf{B}_x :

$$\mathbf{x}_t = \mathbf{A}_x \mathbf{x}_{t-1} + \mathbf{B}_x \varepsilon_t^k. \quad (3)$$

Note that I allow the vector of residuals to depend on k as the noise in the public information may vary across periods.

Next, I guess the time t stock price –corresponding to k periods after the last dividend payment– to be

$$P_t^k = p^k + \mathbf{p}_i^k \mathbf{x}_t + \mathbf{p}_u^k \hat{\mathbf{x}}_t, \quad (4)$$

where $\hat{\mathbf{x}}_t = E_t^u[\mathbf{x}_t]$. Uninformed investors learn $\Pi^k \equiv \mathbf{p}_i^k \mathbf{x}_t$ from observing prices which implies

$$\mathbf{p}_i^k \mathbf{x}_t = E_t^u(\mathbf{p}_i^k \mathbf{x}_t) = \mathbf{p}_i^k \hat{\mathbf{x}}_t.$$

Hence, one of the variables in $\hat{\mathbf{x}}_t$ can be expressed as a linear function of other variables and thus drops from the price function. I drop \hat{Z}_t by noting that

$$\hat{Z}_t = \frac{1}{p_{i2}} \mathbf{p}_i \mathbf{x}_t - \frac{\mathbf{p}_i \mathbf{I}_{-2}}{p_{i2}} \hat{\mathbf{x}}_t, \quad (5)$$

where p_{i2} is the second element of \mathbf{p}_i , and \mathbf{I}_{-2} conforms with the state vector and denotes the matrix that is the same as the identity matrix, except that the (2, 2) element equals zero.

Thus, I set $p_{u2} = 0$.

The next proposition characterizes the equilibrium price function.

Proposition 1 *In equilibrium, the price function is given by,*

$$\begin{aligned} P_t^k &= p^k + \frac{R^k \rho_F^{K+1-k}}{R^{K+1} - \rho_F^{K+1}} F_t + R^{-(K+1-k)} \sum_{j=0}^{k-1} \varepsilon_{t-j}^D \\ &\quad + p_{i2}^k Z_t - p_{u1}^k (F_t - \hat{F}_t) - \sum_{j=0}^{k-1} p_{u3+j}^k (\varepsilon_{t-j}^D - \hat{\varepsilon}_{t-j}^D), \end{aligned} \quad (6)$$

for any $k = 0, \dots, K$. The price function is such that $p^k < 0$, and $p_{i2}^k < 0$ if and only if $\text{Cov}_t^i(Q_{t+1}, q_{t+1}) > 0$.

The first line of the price function describes the present value of dividends when all information is public. The present value calculation accounts for the fact that at time (t, k) , it takes $K + 1 - k$ periods until dividends are paid. The persistent shock ε_t^F dies out over time according to ρ_F , and continues to matter for pricing even after the next dividend payment. The transitory shock ε^D enters the stock price function because (some) investors learn about it before being paid as dividend. Thus, despite being transitory ε^D has de facto persistence of one until the next dividend payment and persistence of zero thereafter.

The second line has two components. The first is the effect of rebalancing by informed investors: When the two assets are positively correlated an increase in the expected return of the private investment opportunity Z_t shifts some of the weight that informed investors put on the stock onto the private investment, thus lowering the stock price. This aspect of the stock price can be interpreted as the stock's ability to absorb liquidity shocks in which case p_{i2}^k describes the level of liquidity in the stock. A period k with low (i.e., less negative) p_{i2}^k can be interpreted as a high liquidity period.

The second component describes the effect of asymmetric information. With $p_{uj}^k \geq 0$, knowing that the stock is better than uninformed investors think it is, i.e., $F_t - \hat{F}_t > 0$ or $\varepsilon_{t+j}^D - \hat{\varepsilon}_{t+j}^D > 0$, gives informed investors knowledge that the price is lower than it should be if it is to reflect the present discounted value of dividends.

3.2 Uninformed investors' filtering problem

Over time, the quantity of information available to uninformed investors changes, namely to reflect the fact that when $k = 0$ dividends are paid and may be used to forecast the ability to pay dividends in the future. Specifically, at every non-dividend-paying period (t, k) , uninformed investors observe the vector $\mathbf{y}_t^k = [\Pi_t^k, S_t^{Fk}, S_t^{Dk}]^\top$, whereas at dividend-paying periods $(t, 0)$, uninformed investors observe $\mathbf{y}_t^0 = [\Pi_t^0, S_t^{F0}, S_t^{D0}, D_t]^\top$. The appendix shows that it is possible to write the vector of observables making use of the time-varying matrices \mathbf{A}_y^k and \mathbf{B}_y^k ,

$$\mathbf{y}_t^k = \mathbf{A}_y^k \mathbf{x}_t + \mathbf{B}_y^k \varepsilon_t^k, \quad k = 0, \dots, K.$$

Define the conditional volatilities:

$$\Sigma_{xx}^k = \mathbf{B}_x \Sigma_{\varepsilon\varepsilon}^k \mathbf{B}_x^\top, \quad \Sigma_{yy}^k = \mathbf{B}_y \Sigma_{\varepsilon\varepsilon}^k \mathbf{B}_y^\top,$$

and $\mathbf{\Omega}_t = E_t^u [(\mathbf{x}_t - \hat{\mathbf{x}}_t)(\mathbf{x}_t - \hat{\mathbf{x}}_t)^\top]$. The next proposition gives the solution to uninformed investors' filtering problem.

Proposition 2 *The steady state Kalman filter is given by the $K + 1$ matrices $\{\Omega^k\}_{k=0,\dots,K}$ that recursively solve the system of $K + 1$ Riccati equations:*

$$\begin{aligned}\Omega^k &= \left(\mathbf{I} - \mathbf{K}_k^{k-1} \mathbf{A}_y^k\right) \left(\mathbf{A}_x \Omega^{k-1} \mathbf{A}_x^\top + \Sigma_{xx}^k\right) \\ \mathbf{K}_k^{k-1} &= \left(\mathbf{A}_x \Omega^{k-1} \mathbf{A}_x^\top + \Sigma_{xx}^k\right) \mathbf{A}_y^{k\top} \left(\mathbf{A}_y^k \left(\mathbf{A}_x \Omega^{k-1} \mathbf{A}_x^\top + \Sigma_{xx}^k\right) \mathbf{A}_y^{k\top} + \Sigma_{yy}^k\right)^{-1}.\end{aligned}$$

If t is a dividend paying period and $k = 0$, it should be understood that $k - 1$ stands for K . Uniformed investors forecast of the state vector is given by

$$\hat{\mathbf{x}}_t = \mathbf{A}_x \hat{\mathbf{x}}_{t-1} + \mathbf{K}_k^{k-1} \hat{\boldsymbol{\varepsilon}}_t^k, \quad (7)$$

and

$$\mathbf{y}_t^k = \mathbf{A}_y^k \mathbf{A}_x \hat{\mathbf{x}}_{t-1} + \hat{\boldsymbol{\varepsilon}}_t^k. \quad (8)$$

The residual $\hat{\boldsymbol{\varepsilon}}_t^k = \mathbf{y}_t^k - E_{t-1}^u[\mathbf{y}_t^k]$ is normally distributed with mean zero and covariance matrix $\text{Var}_{t-1}^u(\hat{\boldsymbol{\varepsilon}}_t^k)$ given in the appendix.

I show below that the modeling feature of periodic dividends, which guarantees that the observation vector is time-varying, is sufficient to generate conditional heteroskedasticity in returns even in the absence of asymmetric information. An additional source of time-variation in stock return volatility results from conditional variation in Ω^k due to endogenous changes in the level of asymmetric information.

3.3 Optimal asset allocation

From the optimization problem of both investors I derive the optimal holdings for the stock for each investor,

$$\theta_t^i = \frac{E_t^i [Q_{t+1}^{k+1}]}{\gamma (\sigma_{Q,k}^i)^2 \left(1 - (\rho_{Qq,k}^i)^2\right)} - \frac{\rho_{Qq,k}^i E_t^i [q_{t+1}]}{\gamma \sigma_{Q,k}^i \sigma_q^i \left(1 - (\rho_{Qq,k}^i)^2\right)}, \quad (9)$$

$$\theta_t^u = \frac{E_t^u [Q_{t+1}^{k+1}]}{\gamma (\sigma_{Q,k}^u)^2}. \quad (10)$$

Demand for the stock is composed of a myopic term for both investors. In addition, informed investors may trade the stock to hedge the risk in other components of their portfolio. The hedging demand term, reflects the fact that when the stock is positively correlated with the

private investment, i.e., $\rho_{Qq}^i > 0$, buying more of both assets increases portfolio risk and is undesirable.

To construct the demands, I calculate the expected return of the various assets under the different information sets. Using equation (6), the appendix shows that

$$E_t^i [Q_{t+1}^{k+1}] = e_0^{k+1} + e_{i2}^{k+1} Z_t - e_{u1}^{k+1} (F_t - \hat{F}_t) - \sum_{j=0}^{k-1} e_{uj+3}^{k+1} (\varepsilon_{t-j}^D - \hat{\varepsilon}_{t-j}^D), \quad (11)$$

and

$$E_t^u [Q_{t+1}^{k+1}] = e_0^{k+1} + e_{i2}^{k+1} \hat{Z}_t. \quad (12)$$

Finally, the expected return for the private investment opportunity is $E_t^i [q_{t+1}] = Z_t$. Substituting these expressions in the asset demands gives the next result.

Proposition 3 *The equilibrium stock demand functions are*

$$\theta_t^i = f_{i0}^{k+1} + f_{i1}^{k+1} Z_t + f_{i2}^{k+1} (F_t - \hat{F}_t) + \sum_{j=0}^{k-1} f_{uj+3}^{k+1} (\varepsilon_{t-j}^D - \hat{\varepsilon}_{t-j}^D), \quad (13)$$

$$\theta_t^u = f_{u0}^{k+1} + f_{u1}^{k+1} \hat{Z}_t, \quad (14)$$

where $f_{i0}^{k+1}, f_{u0}^{k+1} > 0$, and $f_{u1}^{k+1} > 0$ and $f_{i1}^{k+1} < 0$ if and only if $Cov_t^i(Q_{t+1}, q_{t+1}) > 0$.

These stock demands guarantee stock market clearing. In equilibrium, uninformed investors trade only to accommodate what they think are rebalancing trades by informed investors as dictated by movements in \hat{Z}_t : Uninformed investors buy when they perceive that informed investors are selling for liquidity reasons (i.e., \hat{Z}_t is large), thus expecting a positive return (see equation (12)). Informed investors trade the stock to rebalance their portfolio by reducing their holdings when private opportunities abound, and to make use of their private information by increasing their holdings when uninformed investors underestimate either F_t or ε_{t-j}^D . As can be seen from equation (11) and the fact that $e_{i2}^{k+1} > 0$, informed investors sell the stock in response to a rebalancing shock, notwithstanding the high expected returns: The expected loss of high future returns on the stock sold comes at the benefit of hedging some of the risk from taking a larger position in the private investment opportunity.

4 Stock Return and Trading Volume Dynamics

This section analysis the dependence of the distribution of stock returns and trading activity on k , the parameter which identifies the distance from the next dividend-paying period.

4.1 The role of periodicity in dividends

To isolate the role of the assumption of periodic dividends, consider a benchmark economy with a representative investor who trades only the stock and has information \mathcal{I}_t^i . In this economy the equilibrium is characterized by a price function identical to that in equation (6) with $p_{i2}^k = p_{uj}^k = 0$ (see the appendix). Further, the following are moments of the stock return conditional on k alone,

$$E_k \left[Q_{t+1}^{k+1} \right] = \gamma \text{Var}_k \left(Q_{t+1}^{k+1} \right), \quad (15)$$

with

$$\text{Var}_k \left(Q_{t+1}^{k+1} \right) = \left(\frac{R^{k+1} \rho_F^{K-k}}{R^{K+1} - \rho_F^{K+1}} \right)^2 \sigma_F^2 + R^{-2(K-k)} \sigma_D^2. \quad (16)$$

The main result from this benchmark model states that conditional stock return volatility increases with k despite the fact that the news released at any period (t, k) displays constant volatility, i.e., the unconditional volatilities associated with ε_t^F and ε_t^D are constant and equal to σ_F^2 and σ_D^2 , respectively. Intuitively, ε_t^F and ε_t^D are only paid as dividends in period $t + K + 1 - k$, and their impact on the stock price at t reflects the necessary discounting. For example, ε_t^D enters the price function at time t with a coefficient of $R^{-(K-k)}$, whereas ε_{t+1}^D enters the price function at time $t + 1$ with a coefficient of $R^{-(K-k-1)} > R^{-(K-k)}$. This means that the impact of news on stock return volatility is also discounted, but less so as k increases. In addition, it is easy to show that discounting also implies that the conditional volatility is convex in k .

In this benchmark model of a representative investor with myopic asset demand, equation (15) states that the expected stock return is proportional to the conditional stock return volatility. Therefore, expected returns increase monotonically, and are convex in k . These properties have implications for the distribution of expected returns. In particular, lower values of expected returns occur for smaller k and are relatively closer together than higher values of conditional returns, which tend to occur more spaced out. The intuition is that news that occur long before dividend payments occur are highly discounted and contribute little to risk, giving rise to periods of concentrated low expected returns. News that occur close to dividend payments impact prices more and contribute more to risk, giving rise to peaks of volatility around dividend announcements when expected returns are also high. This asymmetry in expected returns means that the distribution of expected returns is positively skewed and, as I will show below, is an important determinant of skewness in stock returns.

4.2 The added effect from rebalancing trades

In this subsection, I extend the benchmark model above to allow heterogeneity in investors' investment opportunity sets, while maintaining the assumption of identical information sets.³ In the presence of other correlated assets (see (1)), informed investors trade to rebalance their portfolio giving rise to non-trivial volume properties. The appendix shows that the equilibrium is described by a price function identical to that in equation (6) with $p_{uj}^k = 0$ and $p_{i2}^k < 0$ for all k , from which I can derive the equilibrium conditional mean returns and variance:

$$E_k \left[Q_{t+1}^{k+1} \right] = \gamma \text{Var}_k \left(Q_{t+1}^{k+1} \right) \frac{1 - \rho_{Qq,k}^2}{\lambda + (1 - \lambda) \left(1 - \rho_{Qq,k}^2 \right)}, \quad (17)$$

with

$$\text{Var}_k \left(Q_{t+1}^{k+1} \right) = \left(\frac{R^{k+1} \rho_F^{K-k}}{R^{K+1} - \rho_F^{K+1}} \right)^2 \sigma_F^2 + R^{-2(K-k)} \sigma_D^2 + \left(p_{i2}^{k+1} \right)^2 \sigma_Z^2. \quad (18)$$

The conditional covariance of stock returns with private investment returns is

$$\text{Cov}_{Qq,k} = R^{-(K-k)} \sigma_{Dq}. \quad (19)$$

Combining (10) with (17), I obtain,

$$E_k \left[\theta_t^u \right] = \frac{1 - \rho_{Qq,k}^2}{\lambda + (1 - \lambda) \left(1 - \rho_{Qq,k}^2 \right)}. \quad (20)$$

Investors shift their asset holdings to reflect their subjective risk exposures. Specifically, a high (squared) correlation $\rho_{Qq,k}^2$ means that informed investors can hedge more of the risk associated with the stock and wish to hold more of it.

The presence of the private investment opportunity produces the following three new effects on conditional moments. First, the returns of the stock and private investment are correlated and their covariance increases over time (see equation (19)). The intuition for the increasing covariance is that at time (t, k) the contribution of the risky dividend cash flow to the stock price, which can be hedged using ε_{t+1}^q , is a discounted $R^{-(K-k)} \varepsilon_{t+1}^D$.

Second, holding fixed the volatility of returns, expected returns decrease relative to (15) provided $\rho_{Qq,k}^i \neq 0$. Intuitively, a fraction λ of the investors now perceives the stock to be less risky and only price the risk that cannot be hedged with the private investment opportunity, which is effectively equivalent to a model with lower risk aversion. Third, the conditional

³For consistency, I maintain the label "informed" investors even though in this model all investors share the same information set.

stock return variance increases relative to (16) by the term $(p_{i2}^k)^2 \sigma_Z^2$. This term is due to liquidity shocks that affect prices via informed investors' portfolio rebalancing.

To proceed, I resort to numerical methods because the model does not lend itself to a complete analytical solution. The parameters chosen do not represent a proper calibration of the model but illustrate qualitative patterns that are found to be robust. I focus on the moments discussed above and also on the conditional mean trading volume, $E_k [Vol_t]$. Trading volume is defined as $Vol_t = (1 - \lambda) |\theta_t^u - \theta_{t-1}^u| = (1 - \lambda) |\Delta\theta_t^u|$, and its conditional mean is

$$E_k [Vol_t] = (1 - \lambda) \sqrt{\frac{2}{\pi}} \sigma_{\Delta\theta_t^u, k}.$$

Mean trading volume is derived from the fact that volume has a Chi distribution with one degree of freedom. Mean trading volume changes with the volatility of net acquisitions by uninformed investors, $\sigma_{\Delta\theta_t^u, k}^2$. The appendix shows how to compute $\sigma_{\Delta\theta_t^u, k}^2$.

Figure 1 illustrates some equilibrium properties under two scenarios, low versus high σ_Z^2 . For sufficiently low values of σ_Z^2 , the rebalancing effect is of second order for the variance of returns and the last term in (18) is small. It helps to consider the limiting case of $\sigma_Z^2 \rightarrow 0$. The conditional return volatility equals that in the benchmark model, is increasing with k , and, it can be easily shown, it increases with k faster than the conditional covariance. Thus, the correlation of the stock return with the private investment return declines monotonically with k . The stock becomes an increasingly poor hedging asset for informed investors' private investment as k increases, and their stock holdings decrease with k (see equation (20)). To encourage uninformed investors to buy at an increasing rate there must be an increasing dispersion in expected returns at high values (even relative to the dispersion in conditional variance values), generating positive skewness in expected returns over and above the skewness predicted by the periodicity of dividends.

When $\sigma_Z^2 > 0$, the effect described above is complemented with a liquidity effect acting via the conditional volatility of returns. As σ_Z^2 increases, the liquidity effect becomes the main driver of the conditional variance of returns because the weight of the term $(p_{i2}^{k+1})^2 \sigma_Z^2$ on total variance increases. In particular, I find numerically that the new variance term is relatively flat with respect to k compared to the monotonicity of variance induced by the periodic nature of dividends.⁴ On the one hand, this leads to a less skewed distribution of

⁴For sufficiently small σ_Z^2 , I find numerically that the liquidity effect may increase the skewness in $\sigma_{Q, k}^2$, thus increasing skewness in conditional expected returns. This effect may dominate the effect of liquidity through ρ_{Qq} for very small values of σ_Z^2 .

conditional variance of returns and, by (17), a less skewed distribution of expected stock returns. On the other hand, given that the conditional covariance of the stock return with the private investment return is increasing in k (see (19)), the correlation $\rho_{Qq,k}$ becomes increasing with k as well.⁵ This gives informed investors an incentive to buy increasingly more of the stock on average as k increases, but can only be an equilibrium outcome if uninformed investors, which provide liquidity, are compensated by selling at high prices well before the dividend announcement. Both effects suggest a flattening of expected returns prior to the dividend announcement leading to lower skewness in expected returns (see Figure 1). The presence of liquidity shocks thus offsets the increase in expected returns prior to dividend announcements caused by the periodicity of dividends.

Rebalancing trades lead to non-trivial trading volume as seen in Figure 1. The main difference between the low and high σ_Z^2 cases regarding trading volume is that in the former, conditional trading volume is decreasing in k , whereas in the later it is increasing in k . This difference is caused by the patterns of the conditional correlation of stock returns and private investment returns. When the conditional correlation increases with k , informed investors' hedging demand introduces increasing variation in stock holdings in response to the volatility in the expected private investment return, i.e., σ_Z^2 .

4.3 The complete model

In the complete model, investors differ also in their information sets, \mathcal{I}_t^i and \mathcal{I}_t^u . Information asymmetry introduces a third effect into conditional moments. Namely, despite some revelation of information via the price, informed investors may accumulate private information as k increases thus reducing the risk they face when holding the stock and increasing their expected return.

With asymmetric information the expected return becomes,

$$E_k(Q_{t+1}^{k+1}) = \frac{\gamma (\sigma_{Q,k}^i)^2 \left(1 - (\rho_{Qq,k}^i)^2\right) (\sigma_{Q,k}^u)^2}{\lambda (\sigma_{Q,k}^u)^2 + (1 - \lambda) (\sigma_{Q,k}^i)^2 \left(1 - (\rho_{Qq,k}^i)^2\right)}. \quad (21)$$

As before, the expected return increases with risk aversion and with increases in the risk of the stock as perceived by either informed or uninformed investors. Combining (10) with (21),

⁵To summarize the monotonicity properties of $\rho_{Qq,k}^2$ with respect to k , it decreases with k for low σ_Z^2 , but it increases in k for sufficiently large σ_Z^2 .

I obtain the conditional mean holdings of uninformed investors,

$$E_k [\theta_t^u] = \frac{(\sigma_{Q,k}^i)^2 \left(1 - (\rho_{Qq,k}^i)^2\right)}{\lambda (\sigma_{Q,k}^u)^2 + (1 - \lambda) (\sigma_{Q,k}^i)^2 \left(1 - (\rho_{Qq,k}^i)^2\right)}. \quad (22)$$

In an equilibrium where informed investors accumulate private information as k increases, their conditional return variance decreases with k and uninformed investors' conditional mean holdings decrease.

It is not possible to obtain a (quasi-) closed form solution to the conditional variance of stock returns, $\sigma_{Q,k}^2 = \text{Var}_k \left(Q_{t+1}^{k+1}\right)$. The conditional variance differs from those in the previous models in that asymmetric information changes the way shocks affect prices and also introduces additional volatility via the forecast errors of uninformed investors.

To analyze the properties of the model, I evaluate the equilibrium numerically in the case of maximal asymmetry of information, $\sigma_{Fk}^2, \sigma_{Dk}^2 \rightarrow \infty$. Figure 2 presents the equilibria that result under two scenarios, small and large σ_Z^2 .⁶ The two cases produce similar, qualitative implications for the conditional return and variance and for conditional mean holdings of uninformed investors. These implications are also quite similar to those in Figure 1 for the case of large σ_Z^2 . The presence of asymmetric information generates an incentive for informed investors to buy more of the stock as the date of the dividend payment nears. This is because the information asymmetry is highest just before the dividend is paid, even after accounting for information revealed through the market price. This information effect magnifies the rebalancing effect associated with the monotonicity of $\rho_{Qq,k}$ that exists when σ_Z^2 is high. When σ_Z^2 is low, it may dominate both the effect associated with periodic dividends and the rebalancing effect which would otherwise lead to a declining ρ_{Qq} . Therefore, expected returns display more of an hump-shape prior to dividend announcements, leading to increased negative skewness.

What makes the case of low σ_Z^2 stand out is its implications for mean volume. When σ_Z^2 is low, information trades tend to dominate rebalancing trades worsening the adverse selection problem. As informed investors' private information accumulates with k , the adverse selection problem becomes more acute, and trading volume dries up prior to dividend announcements.

⁶In Figure 2 I use a lower value for σ_Z^2 than in Figure 1 since otherwise the juxtaposition of the plots would not allow for a clear observation of the various patterns.

5 Skewness, Asymmetric Information, and Liquidity

Conditionally, equilibrium excess stock returns are normally distributed. Unconditionally, however, they are not normal because the mean and variance of a randomly drawn return observation depend on k . Hence, because a k -period stock return is drawn from a normal density $\phi(Q; e_0^k, \sigma_{Q,k}^2)$ and such observations occur with frequency $1/(K+1)$, the unconditional distribution of returns is a mixture of normals distribution. Formally,

Proposition 4 *The unconditional distribution of stock returns is a mixture of normals distribution with density*

$$f(Q) = \frac{1}{K+1} \sum_{k=0}^K \phi(Q; e_0^k, \sigma_{Q,k}^2),$$

where $\phi(\cdot)$ is the normal density function.

The periodicity of dividends, by generating time-varying conditional volatility in stock returns, leads to the derived mixture of normals distribution for stock returns for $K \geq 1$. This result provides a theoretical justification for attempting to fit a mixture of normals distribution for stock returns (e.g., Kon, 1984).

In the appendix, I prove the following corollary.

Corollary 1 *The unconditional mean and variance of stock returns are*

$$\begin{aligned} E(Q_{t+1}) &= \frac{1}{K+1} \sum_{k=0}^K e_0^k, \\ \text{Var}(Q_{t+1}) &= \frac{1}{K+1} \sum_{k=0}^K \left[\sigma_{Q,k}^2 + \left(e_0^k - E(Q_{t+1}) \right)^2 \right]. \end{aligned}$$

The unconditional skewness in stock returns is

$$E \left[(Q - E(Q_{t+1}))^3 \right] = \frac{1}{K+1} \sum_{k=0}^K \left(e_0^k - E(Q_{t+1}) \right)^3 + \frac{3}{(K+1)^2} \sum_{k=0}^K \sum_{j < k} (\sigma_{Q,k}^2 - \sigma_{Q,j}^2) (e_0^k - e_0^j). \quad (23)$$

The unconditional mean return is simply the mean of the k -conditional expected returns, $e_0^k = E_k(Q_{t+1}^{k+1})$. The unconditional mean variance is an average of the k -conditional variances plus the mean of squared deviations of each of the k -conditional means to the unconditional mean.

Skewness in stock returns can be broken down into two parts. The first term in (23) is the level of skewness in expected returns, e_0^k . This term is positive (negative) when a small number of expected returns display high (small) values. The second term describes the impact on skewness of the correlation of return volatility with expected returns. Loosely speaking, this second term is related to the leverage and feedback hypotheses that entertain a negative correlation between current realized returns (hence high expected returns) and high conditional volatility. The literature that exploits asymmetric volatility as a driver of skewness thus focuses on this term. When returns follow a mixture of normals, unconditional skewness does not depend on the correlation between *realized* returns and conditional volatility. Moreover, skewness in expected returns also matters for overall skewness in stock returns.

When $K = 0$, returns are unconditionally normally distributed and skewness is zero. When $K = 1$, because each k -period is weighted equally, it is straightforward to show that $\sum_{k=0}^1 (e_0^k - E(Q_{t+1}))^3 = 0$. Skewness is

$$E \left[(Q - E(Q_{t+1}))^3 \right] = \frac{3}{4} (\sigma_{Q,0}^2 - \sigma_{Q,1}^2) (e_0^0 - e_0^1). \quad (24)$$

When $K = 1$, positive skewness is thus an implication of a risk-return trade off; when periods of high expected returns are associated with periods of high volatility. Furthermore, the stronger the risk-return trade off, the higher is the skewness. In contrast, the ability to predict negative skewness is likely to represent a challenge to theories that predict both that returns follow a mixture of normals distribution and a risk-return trade off.

When $K > 1$, the first term in (23) is no longer necessarily zero and skewness in expected returns also matters. Thus, unconditional skewness in returns becomes harder to sign. However, it remains that the second term in (23) is positive when $\sigma_{Q,k}^2 > \sigma_{Q,j}^2$ and $e_0^k > e_0^j$. Therefore, while a risk-return trade off tends to generate positive skewness, it is no longer a necessary nor a sufficient condition.

5.1 Periodicity of dividends and skewness

To understand the various drivers of skewness, consider again the benchmark model of subsection 4.1. When $K = 1$, skewness can be easily derived by first combining (24) with (15)

and then using (16) to get:

$$\begin{aligned} E \left[(Q - E(Q_{t+1}))^3 \right] &= \frac{3\gamma}{4} (\sigma_{Q,0}^2 - \sigma_{Q,1}^2)^2 \\ &= \frac{3\gamma}{4} \left(\frac{R^2}{R^2 - \rho_F^2} \sigma_F^2 + \frac{R^2 - 1}{R^2} \sigma_D^2 \right)^2. \end{aligned}$$

Skewness increases with risk aversion γ . Higher risk aversion increases the sensitivity of expected returns to conditional volatility and thus leads to higher positive skewness. Skewness also increases with volatility of dividends as given by σ_F^2 and σ_D^2 , because volatility has a level effect on $\sigma_{Q1}^2 - \sigma_{Q0}^2$ and hence on skewness. Intuitively, increases in volatility magnify the dispersion in volatility induced by discounting, leading to a stronger risk-return trade off. When $K = 2$, one must also factor in skewness in expected returns as discussed above. I show in the appendix that skewness is also positive and varies in the same way with those parameters.⁷

This benchmark model is consistent with two stylized facts about firm-level skewness in the US and abroad. First, firm-level skewness in daily returns is generally positive (see Kon, 1984, Chen et al., 2001, and Bae et al., 2006). Second, firm-level skewness increases with firm volatility. In Chen et al. (2001), firm-level skewness is shown to increase with firm-level return volatility and decrease with firm size. Bae et al.(2006) also show that firm-level skewness decreases with firm size. If smaller firms have more volatile cash flows, then these results suggest that indeed firms with more volatile cash flows have higher skewness. While many empirical studies control for volatility as a driver of skewness, there are no theories for why firms with more volatility should have more positively skewed returns. In fact, as Chen et al. (2001) argue, the leverage and feedback theories of skewness would tend to predict a negative association between volatility and skewness.

5.2 Liquidity shocks and skewness

Consider the model of subsection 4.2 again, now to analyze the effects of liquidity shocks on skewness. Recall that an increase in σ_Z^2 tends to generate lower and possibly negative skewness in expected returns, which contributes to lower skewness in stock returns. In addition, liquidity shocks also impact the risk-return trade off. The main channel is through a decrease in the skewness of conditional volatility of returns for large σ_Z^2 , which decreases

⁷For $K \geq 3$, calculations get very messy quickly and I am not able to show that skewness is always positive. However, the result above that the conditional expected return is increasing and convex in k , (and numerical simulations,) suggests a concentration of conditional expected returns at low values, and thus positive skewness.

the risk return trade off. For low σ_Z^2 , liquidity shocks may amplify the dispersion in the conditional volatility, leading to greater skewness in stock returns. Figure 3 suggests that by and large the former two effects dominate, inducing a negative association between liquidity shocks and skewness in stock returns.

The monotonicity of unconditional mean volume, $(K + 1)^{-1} \sum_k E_k [Vol_t]$, with respect to σ_Z^2 , depends on the relative strength of two factors. First, σ_Z^2 has a direct positive effect on the volatility of holdings through the volatility of the conditional private investment return. This is the traditional effect of liquidity shocks. Second, σ_Z^2 has an indirect negative effect through ρ_{Qq} which declines as σ_Z^2 increases, discouraging overall rebalancing by informed investors. Figure 3 shows that the former dominates for low values of σ_Z^2 , whereas the later effect dominates for high values of σ_Z^2 .

Variation in the relative size of liquidity shocks can thus produce a negative association between skewness and turnover at low values of σ_Z^2 , while generating positive skewness in returns. In this range, liquidity shocks increase turnover. At the same time, liquidity shocks decrease the skewness in expected returns because informed investors become increasingly eager to hold relatively more of the stock close to a dividend announcement and encourage uninformed to sell (or to buy less) by bidding up the price prior to the dividend announcement and flattening subsequent expected returns.

This prediction is consistent with the evidence in Chen et al. (2001) that (i) high turnover is associated with lower skewness, and (ii) unconditional mean skewness is positive. In Hong and Stein (2003), high turnover is also associated with lower skewness, but unconditional mean skewness is negative. In their model, if bearish investors cannot short the stock and chose to leave the market, their information is not incorporated into prices. When more negative information arises which makes other investors also bearish, the cumulative effect of the new information and of the old information is a sharp decline in prices which may be associated with high volatility and high trading volume.

5.3 Asymmetric information and skewness

Consider again the complete model where agents are heterogeneous also with respect to their information sets. I start by first illustrating the effects of liquidity on skewness in this model, and then move on to discuss the effects of information asymmetry.

Figure 4 plots skewness and turnover for various levels of the liquidity shock σ_Z^2 assuming maximal asymmetry of information, $\sigma_{Fk}^2, \sigma_{Dk}^2 \rightarrow \infty$. As σ_Z^2 increases, informed investors are

better able to mask their information trades and less information is revealed through the price. This gives informed investors an extra incentive to hold more of the stock as the dividend payment approaches. Hence, they must bid the stock price up long before that in order to buy from uninformed investors. Expected returns have a pronounced hump-shape and are negatively skewed contributing to overall negative skewness in stock returns. Skewness in expected returns is the dominant source of variation in skewness, eventually generating negative skewness for large σ_Z^2 .

Turnover is increasing in σ_Z^2 over a wider range of values for σ_Z^2 relative to the model without asymmetric information. The reason is that in the presence of asymmetric information, larger liquidity shocks contribute to more turnover by decreasing uninformed investors' adverse selection problem. Thus, the model with asymmetric information also predicts a negative association between turnover and skewness.

The effect of asymmetric information on skewness and turnover is depicted in Figure 5. In the figure, I allow σ_{Dk}^2 and σ_{Fk}^2 to vary from 0.01 (low information asymmetry) to 0.96 (high information asymmetry). The increase in information asymmetry, for fixed σ_Z^2 , leads to lower skewness as suggested above. For turnover, the increase in information asymmetry produces a non-monotonic pattern. A negative association is to be expected when the adverse selection effect is strong enough. A positive association may arise if the level of liquidity shocks is sufficiently large. To see this note that when $\sigma_{Dk}^2 = \sigma_{Fk}^2 \rightarrow 0$ only rebalancing trades exist. For small σ_{Dk}^2 and σ_{Fk}^2 , trading volume may increase with noise in public news because informed investors can and will exploit their information advantage by trading on it besides trading for rebalancing reasons.

The model with asymmetric information predicts a negative association between skewness and volume caused by changes in the level of information asymmetry. In addition to predicting this negative relation, the model provides two testable hypotheses that differ from those in Hong and Stein (2003). One is that the negative association occurs even though skewness is positive. In Hong and Stein, skewness is predicted to be negative. The other is that, the negative association is predicted only for low levels of information asymmetry.⁸ In Hong and Stein, it requires large difference in opinions. The evidence strongly supports that firm level stock returns display positive skewness. It is up to further testing determining how investor "disagreement" impacts the association between skewness and turnover.

⁸In the extreme case of no information asymmetry, liquidity shocks can also predict the same negative association (see subsection 5.2).

6 Conclusion

This paper analyzes the asset pricing implications of periodicity in dividend payments within the context of a stationary rational expectations model with heterogeneous investors. The paper establishes that periodic dividends gives rise to time-varying conditional volatility in stock returns and is an underlying source for positive skewness in the model. The periodicity of dividends is sufficient to predict that the unconditional distribution of returns is a mixture of normals distribution.

The paper also demonstrates the effects of liquidity shocks and information asymmetry on conditional moments and skewness. I show that skewness in expected returns may be an important driver of skewness in stock returns. I also show that both liquidity shocks and asymmetric information may cause a negative association between skewness in stock returns and turnover.

Future theoretical research would aim to understand related sources of time variation in conditional moments in stock returns, including the fact that dividend announcements are made with a lag relative to the actual dividend payment. In addition, not all news events are associated with dividend news. Separately identifying the effect of public cash flow news from dividend announcements is also left for future research, possibly by enriching the model of dividend changes. Future empirical research should try to determine how the level of investor disagreement affects the negative association between firm-level skewness and volatility.

Appendix

A Proofs

I start by proving Proposition 2 and then turn to the proofs of propositions 1 and 3. The proofs of propositions 1-3 follow the general approach developed in Albuquerque and Miao (2009).

Proof of Proposition 2: The matrices that describe the state vector dynamics in equation (3) are given by

$$\mathbf{A}_x = \begin{bmatrix} \rho_F & 0 & \dots & & 0 \\ 0 & \rho_Z & 0 & \dots & 0 \\ 0 & 0 & 0 & \dots & 0 \\ 0 & 0 & 1 & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & 0 \\ 0 & 0 & \dots & 0 & 1 & 0 \end{bmatrix}, \quad \mathbf{B}_x = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & \dots & 0 & & & \\ & 0 & \dots & 0 & & \\ & & & 0 & \dots & 0 & 0 \end{bmatrix},$$

and the conditional covariance matrix of the vector of residuals when period t is k periods after the last dividend payment is,

$$\Sigma_{\varepsilon\varepsilon}^k = E[\boldsymbol{\varepsilon}_t^k \boldsymbol{\varepsilon}_t^{k\top}] = \begin{bmatrix} \sigma_F^2 & 0 & 0 & 0 & 0 & 0 \\ & \sigma_{Z^D}^2 & 0 & 0 & 0 & 0 \\ & & \sigma_D^2 & \sigma_{Dq} & 0 & 0 \\ & & & \sigma_q^2 & 0 & 0 \\ & & & & \sigma_{Fk}^2 & 0 \\ & & & & & \sigma_{Dk}^2 \end{bmatrix}.$$

Uninformed investors observe the following signals at any time t :

$$\begin{aligned} \mathbf{y}_t^0 &= [\Pi_t^0 \ S_t^{F0} \ S_t^{D0} \ D_t]^\top = \mathbf{A}_y^0 \mathbf{x}_t + \mathbf{B}_y^0 \boldsymbol{\varepsilon}_t^0, \\ \mathbf{y}_t^k &= [\Pi_t^k \ S_t^{Fk} \ S_t^{Dk}]^\top = \mathbf{A}_y^k \mathbf{x}_t + \mathbf{B}_y^k \boldsymbol{\varepsilon}_t^k, \quad k = 1, \dots, K, \end{aligned}$$

where

$$\begin{aligned} \mathbf{A}_y^0 &= \begin{bmatrix} \mathbf{p}_i^0 \\ \mathbf{c}_1 \\ \mathbf{c}_3 \\ \mathbf{c}_{-2} \end{bmatrix}, \quad \mathbf{A}_y^k = \begin{bmatrix} \mathbf{p}_i^k \\ \mathbf{c}_1 \\ \mathbf{c}_3 \end{bmatrix}, \\ \mathbf{B}_y^0 &= \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad \mathbf{B}_y^k = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}. \end{aligned}$$

The row vectors \mathbf{c}_l have zeros in all columns except at position l . The row vector \mathbf{c}_{-2} has ones everywhere except at column 2.

Define the conditional volatilities:

$$\Sigma_{xx}^k = \mathbf{B}_x \Sigma_{\varepsilon\varepsilon}^k \mathbf{B}_x^\top, \quad \Sigma_{yy}^k = \mathbf{B}_y \Sigma_{\varepsilon\varepsilon}^k \mathbf{B}_y^\top,$$

and $\Omega_t^k = E_t^u[(\mathbf{x}_t - \hat{\mathbf{x}}_t)(\mathbf{x}_t - \hat{\mathbf{x}}_t)^\top]$. The steady state Kalman filter is given by the $K + 1$ matrices (see Anderson and Moore, 1979)

$$\Omega^0, \dots, \Omega^K$$

that solve the Riccati equations

$$\begin{aligned} \Omega_k^k &= \Omega_k^{k-1} - \mathbf{K}_k^{k-1} \mathbf{A}_y^k \Omega_k^{k-1} \\ \mathbf{K}_k^{k-1} &= \Omega_k^{k-1} \mathbf{A}_y^k \left(\mathbf{A}_y^k \Omega_k^{k-1} \mathbf{A}_y^k + \Sigma_{yy}^k \right)^{-1} \\ \Omega_k^{k-1} &= \mathbf{A}_x \Omega_k^{k-1} \mathbf{A}_x^\top + \Sigma_{xx}^k. \end{aligned} \quad (\text{A.1})$$

If t is a dividend paying period, replace Ω_k^{k-1} by Ω_0^K and \mathbf{K}_k^{k-1} by \mathbf{K}_0^K . I then obtain the steady-state filters as described in the proposition. The residual $\hat{\varepsilon}_t^k = \mathbf{y}_t^k - E_{t-1}^u[\mathbf{y}_t^k]$ is normally distributed with mean of zero and conditional covariance matrix

$$\text{Var}_{t-1}(\hat{\varepsilon}_t) = \mathbf{A}_y^k \mathbf{A}_x \Omega_k^{k-1} \mathbf{A}_x^\top \mathbf{A}_y^k + \left(\mathbf{A}_y^k \mathbf{B}_x + \mathbf{B}_y \right) \Sigma_{\varepsilon\varepsilon}^k \left(\mathbf{A}_y^k \mathbf{B}_x + \mathbf{B}_y \right)^\top. \quad (\text{A.2})$$

■

Proof of Proposition 1: Using the guessed price function in (4) write the excess return at time $t + 1$, assuming that $t + 1$ is a non-dividend paying period (i.e., $K - 1 \geq k \geq 0$ at time t), as

$$\begin{aligned} Q_{t+1}^{k+1} &= p^{k+1} + \mathbf{p}_i^{k+1} \mathbf{x}_{t+1} + \mathbf{p}_u^{k+1} \mathbf{I}_{-2} \hat{\mathbf{x}}_{t+1} - R \left(p^k + \mathbf{p}_i^k \mathbf{x}_t + \mathbf{p}_u^k \mathbf{I}_{-2} \hat{\mathbf{x}}_t \right) \\ &= p^{k+1} - R p^k + \left(\mathbf{p}_i^{k+1} \mathbf{A}_x - R \mathbf{p}_i^k \right) \mathbf{x}_t + \left(\mathbf{p}_u^{k+1} \mathbf{I}_{-2} \mathbf{A}_x - R \mathbf{p}_u^k \mathbf{I}_{-2} \right) \hat{\mathbf{x}}_t \\ &\quad + \mathbf{p}_i^{k+1} \mathbf{B}_x \varepsilon_{t+1}^{k+1} + \mathbf{p}_u^{k+1} \mathbf{I}_{-2} \mathbf{K}_{k+1}^k \hat{\varepsilon}_{t+1}^{k+1} \\ &= p^{k+1} - R p^k + \left(\mathbf{p}_i^{k+1} \mathbf{A}_x - R \mathbf{p}_i^k \right) \mathbf{x}_t + \left(\mathbf{p}_u^{k+1} \mathbf{I}_{-2} \mathbf{A}_x - R \mathbf{p}_u^k \mathbf{I}_{-2} \right) \hat{\mathbf{x}}_t + \mathbf{p}_i^{k+1} \mathbf{B}_x \varepsilon_{t+1}^{k+1} \\ &\quad + \mathbf{p}_u^{k+1} \mathbf{I}_{-2} \mathbf{K}_{k+1}^k \left(\mathbf{A}_y^{k+1} \mathbf{A}_x \left(\mathbf{I}_{-2} - \mathbf{c}_2^\top \frac{1}{p_{i2}^k} \mathbf{p}_i^k \mathbf{I}_{-2} \right) (\mathbf{x}_t - \hat{\mathbf{x}}_t) + \left(\mathbf{A}_y^{k+1} \mathbf{B}_x + \mathbf{B}_y^{k+1} \right) \varepsilon_{t+1}^{k+1} \right) \\ &= e_0^{k+1} + \mathbf{e}_i^{k+1} \mathbf{x}_t + \mathbf{e}_u^{k+1} \hat{\mathbf{x}}_t + \mathbf{b}_Q^{k+1} \varepsilon_{t+1}^{k+1} \end{aligned}$$

where the constants

$$e_0^{k+1} = p^{k+1} - Rp^k \quad (\text{A.3})$$

$$\mathbf{e}_i^{k+1} = \mathbf{p}_i^{k+1} \mathbf{A}_x - R\mathbf{p}_i^k + \mathbf{p}_u^{k+1} \mathbf{I}_{-2} \mathbf{K}_{k+1}^k \mathbf{A}_y^{k+1} \mathbf{A}_x \left(\mathbf{I}_{-2} - \mathbf{c}_2^\top \frac{1}{p_{i2}^k} \mathbf{p}_i^k \mathbf{I}_{-2} \right) \quad (\text{A.4})$$

$$\mathbf{e}_u^{k+1} = \mathbf{p}_u^{k+1} \mathbf{I}_{-2} \mathbf{A}_x - R\mathbf{p}_u^k \mathbf{I}_{-2} - \mathbf{p}_u^{k+1} \mathbf{I}_{-2} \mathbf{K}_{k+1}^k \mathbf{A}_y^{k+1} \mathbf{A}_x \left(\mathbf{I}_{-2} - \mathbf{c}_2^\top \frac{1}{p_{i2}^k} \mathbf{p}_i^k \mathbf{I}_{-2} \right) \quad (\text{A.5})$$

$$\mathbf{b}_Q^{k+1} = \mathbf{p}_i^{k+1} \mathbf{B}_x + \mathbf{p}_u^{k+1} \mathbf{I}_{-2} \mathbf{K}_{k+1}^k \left(\mathbf{A}_y^{k+1} \mathbf{B}_x + \mathbf{B}_y^{k+1} \right). \quad (\text{A.6})$$

To arrive at the third equality above, I use the fact that

$$\begin{aligned} \hat{\boldsymbol{\varepsilon}}_{t+1}^{k+1} &= \mathbf{y}_{t+1}^{k+1} - \mathbf{A}_y^{k+1} \mathbf{A}_x \hat{\mathbf{x}}_t \\ &= \mathbf{A}_y^{k+1} \left(\mathbf{A}_x \mathbf{x}_t + \mathbf{B}_x \boldsymbol{\varepsilon}_{t+1}^{k+1} \right) + \mathbf{B}_y^{k+1} \boldsymbol{\varepsilon}_{t+1}^{k+1} - \mathbf{A}_y^{k+1} \mathbf{A}_x \hat{\mathbf{x}}_t \\ &= \mathbf{A}_y^{k+1} \mathbf{A}_x (\mathbf{x}_t - \hat{\mathbf{x}}_t) + \left(\mathbf{A}_y^{k+1} \mathbf{B}_x + \mathbf{B}_y^{k+1} \right) \boldsymbol{\varepsilon}_{t+1}^{k+1} \\ &= \mathbf{A}_y^{k+1} \mathbf{A}_x \left(\mathbf{I}_{-2} - \mathbf{c}_2^\top \frac{1}{p_{i2}^k} \mathbf{p}_i^k \mathbf{I}_{-2} \right) (\mathbf{x}_t - \hat{\mathbf{x}}_t) + \left(\mathbf{A}_y^{k+1} \mathbf{B}_x + \mathbf{B}_y^{k+1} \right) \boldsymbol{\varepsilon}_{t+1}^{k+1}, \end{aligned}$$

where the last equality follows from (5),

$$Z_t - \hat{Z}_t = \frac{1}{p_{i2}^k} \mathbf{p}_i^k \mathbf{I}_{-2} (\hat{\mathbf{x}}_t - \mathbf{x}_t),$$

and from

$$\begin{aligned} \mathbf{x}_t - \hat{\mathbf{x}}_t &= \mathbf{I}_{-2} (\mathbf{x}_t - \hat{\mathbf{x}}_t) + \mathbf{c}_2^\top \left(Z_t - \hat{Z}_t \right) \\ &= \mathbf{I}_{-2} (\mathbf{x}_t - \hat{\mathbf{x}}_t) - \mathbf{c}_2^\top \frac{1}{p_{i2}^k} \mathbf{p}_i^k \mathbf{I}_{-2} (\mathbf{x}_t - \hat{\mathbf{x}}_t) \\ &= \left(\mathbf{I}_{-2} - \mathbf{c}_2^\top \frac{1}{p_{i2}^k} \mathbf{p}_i^k \mathbf{I}_{-2} \right) (\mathbf{x}_t - \hat{\mathbf{x}}_t). \end{aligned}$$

Conditional expected returns equal

$$\begin{aligned} E_t^i \left[Q_{t+1}^{k+1} \right] &= e_0^{k+1} + \mathbf{e}_i^{k+1} \mathbf{x}_t + \mathbf{e}_u^{k+1} \hat{\mathbf{x}}_t \\ E_t^u \left[Q_{t+1}^{k+1} \right] &= e_0^{k+1} + \left(\mathbf{e}_i^{k+1} + \mathbf{e}_u^{k+1} \right) \hat{\mathbf{x}}_t. \end{aligned}$$

Use (5) to substitute out \hat{Z}_t in $E_t^u \left[Q_{t+1}^{k+1} \right]$ (because likely $e_{i2}^{k+1} \neq 0$) and derive

$$\begin{aligned} E_t^u \left[Q_{t+1}^{k+1} \right] &= e_0^{k+1} + \mathbf{e}_i^{k+1} \left[\mathbf{I}_{-2} \hat{\mathbf{x}}_t + \mathbf{c}_2^\top \hat{Z}_t \right] + \mathbf{e}_u^{k+1} \hat{\mathbf{x}}_t \\ &= e_0^{k+1} + \mathbf{e}_i^{k+1} \left[\mathbf{I}_{-2} \hat{\mathbf{x}}_t + \mathbf{c}_2^\top Z_t - \frac{1}{p_{i2}^k} \mathbf{c}_2^\top \mathbf{p}_i^k \mathbf{I}_{-2} (\hat{\mathbf{x}}_t - \mathbf{x}_t) \right] + \mathbf{e}_u^{k+1} \hat{\mathbf{x}}_t \\ &= e_0^{k+1} + \mathbf{e}_i^{k+1} \left(\mathbf{c}_2^\top \mathbf{c}_2 + \frac{1}{p_{i2}^k} \mathbf{c}_2^\top \mathbf{p}_i^k \mathbf{I}_{-2} \right) \mathbf{x}_t + \left[\mathbf{e}_i^{k+1} \left(\mathbf{I}_{-2} - \frac{1}{p_{i2}^k} \mathbf{c}_2^\top \mathbf{p}_i^k \mathbf{I}_{-2} \right) + \mathbf{e}_u^{k+1} \right] \hat{\mathbf{x}}_t \\ &= e_0^{k+1} + \tilde{\mathbf{e}}_i^{k+1} \mathbf{x}_t + \tilde{\mathbf{e}}_u^{k+1} \hat{\mathbf{x}}_t. \end{aligned}$$

Likewise, I proceed in the same fashion to obtain the expression for excess returns when $t + 1$ is a dividend-paying period:

$$\begin{aligned}
Q_{t+1}^0 &= p^0 + \mathbf{p}_i^0 \mathbf{x}_{t+1} + \mathbf{p}_u^0 \mathbf{I}_{-2} \hat{\mathbf{x}}_{t+1} + \mathbf{c}_{-2} \mathbf{x}_{t+1} - R(p^K + \mathbf{p}_i^K \mathbf{x}_t + \mathbf{p}_u^K \mathbf{I}_{-2} \hat{\mathbf{x}}_t) \\
&= p^0 + (\mathbf{p}_i^0 + \mathbf{c}_{-2}) (\mathbf{A}_x \mathbf{x}_t + \mathbf{B}_x \varepsilon_{t+1}^0) + \mathbf{p}_u^0 \mathbf{I}_{-2} (\mathbf{A}_x \hat{\mathbf{x}}_t + \mathbf{K}_0^K \hat{\varepsilon}_{t+1}^0) - R(p^K + \mathbf{p}_i^K \mathbf{x}_t + \mathbf{p}_u^K \mathbf{I}_{-2} \hat{\mathbf{x}}_t) \\
&= p^0 - Rp^K + ((\mathbf{p}_i^0 + \mathbf{c}_{-2}) \mathbf{A}_x - R\mathbf{p}_i^K) \mathbf{x}_t + (\mathbf{p}_u^0 \mathbf{I}_{-2} \mathbf{A}_x - R\mathbf{p}_u^K \mathbf{I}_{-2}) \hat{\mathbf{x}}_t + (\mathbf{p}_i^0 + \mathbf{c}_{-2}) \mathbf{B}_x \varepsilon_{t+1}^0 \\
&\quad + \mathbf{p}_u^0 \mathbf{I}_{-2} \mathbf{K}_0^K \left(\mathbf{A}_y^0 \mathbf{A}_x \left(\mathbf{I}_{-2} - \mathbf{c}_2^\top \frac{1}{p_i^K} \mathbf{p}_i^K \mathbf{I}_{-2} \right) (\mathbf{x}_t - \hat{\mathbf{x}}_t) + (\mathbf{A}_y^0 \mathbf{B}_x + \mathbf{B}_y^0) \varepsilon_{t+1}^0 \right) \\
&= e_0^0 + \mathbf{e}_i^0 \mathbf{x}_t + \mathbf{e}_u^0 \hat{\mathbf{x}}_t + \mathbf{b}_Q^0 \varepsilon_{t+1}^0,
\end{aligned}$$

with the constants e_0^0 , \mathbf{e}_i^0 , \mathbf{e}_u^0 and \mathbf{b}_Q^0 appropriately defined. Expressions for $E_t^i [Q_{t+1}^0]$ and $E_t^u [Q_{t+1}^0]$ are obtained as before.

The volatility of stock returns conditional on \mathcal{I}_t^i is

$$(\sigma_{Q,k}^i)^2 \equiv E_t^i \left[\left(Q_{t+1}^{k+1} - E_t^i [Q_{t+1}^{k+1}] \right)^2 \right] = \mathbf{b}_Q^{k+1} \Sigma_{\varepsilon\varepsilon}^{k+1} (\mathbf{b}_Q^{k+1})^\top,$$

and the volatility of stock returns conditional on \mathcal{I}_t^u is

$$(\sigma_{Q,k}^u)^2 \equiv E_t^u \left[\left(Q_{t+1}^{k+1} - E_t^u [Q_{t+1}^{k+1}] \right)^2 \right] = \mathbf{e}_i^{k+1} \Omega^k (\mathbf{e}_i^{k+1})^\top + \mathbf{b}_Q^{k+1} \Sigma_{\varepsilon\varepsilon}^{k+1} (\mathbf{b}_Q^{k+1})^\top.$$

The volatility of private investment returns conditional on \mathcal{I}_t^i is

$$(\sigma_q^i)^2 \equiv E_t^i \left[(q_{t+1} - E_t^i [q_{t+1}])^2 \right] = \sigma_q^2.$$

Finally, the covariance between stock and private investment returns conditional on \mathcal{I}_t^i is

$$Cov_{Qq,k}^i \equiv E_t^i \left[\left(Q_{t+1}^{k+1} - E_t^i [Q_{t+1}^{k+1}] \right) (q_{t+1} - E_t^i [q_{t+1}]) \right] = \mathbf{b}_Q^{k+1} \Sigma_{\varepsilon\varepsilon}^{k+1} \mathbf{c}_4^\top,$$

where \mathbf{c}_4^\top is a column vector of zeros with 1 in the fourth position. Define $\rho_{Qq,k}^i$ as the conditional correlation between stock and private investment returns conditional on \mathcal{I}_t^i .

It is now possible to solve for the stock demands and find the stock price that clears the market. From the investors problems, the optimal asset demands are as in (9) and (10). Inserting the asset demands into the stock market clearing condition yields

$$\lambda \frac{E_t^i [Q_{t+1}^{k+1}]}{\gamma (\sigma_Q^i)^2 \left(1 - (\rho_{Qq}^i)^2 \right)} - \lambda \frac{\rho_{Qq}^i E_t^i [q_{t+1}]}{\gamma \sigma_Q^i \sigma_q^i \left(1 - (\rho_{Qq}^i)^2 \right)} + (1 - \lambda) \frac{E_t^u [Q_{t+1}^{k+1}]}{\gamma (\sigma_Q^u)^2} = 1.$$

After replacing the conditional expectations with the expressions above, I get the following set of equilibrium conditions:

$$\lambda \frac{e_0^{k+1}}{\gamma (\sigma_{Q,k}^i)^2 \left(1 - (\rho_{Qq,k}^i)^2\right)} + (1 - \lambda) \frac{e_0^{k+1}}{\gamma (\sigma_{Q,k}^u)^2} = 1,$$

$$\frac{\lambda e_i^{k+1}}{\gamma (\sigma_{Q,k}^i)^2 \left(1 - (\rho_{Qq,k}^i)^2\right)} - \frac{\lambda \rho_{Qq,k}^i \mathbf{c}_2}{\gamma \sigma_{Q,k}^i \sigma_{q,k}^i \left(1 - (\rho_{Qq,k}^i)^2\right)} + \frac{(1 - \lambda) e_i^{k+1} \left(\mathbf{c}_2^T \mathbf{c}_2 + \frac{1}{p_{i2}^k} \mathbf{c}_2^T \mathbf{p}_i^k \mathbf{I}_{-2}\right)}{\gamma (\sigma_{Q,k}^u)^2} = 0, \quad (\text{A.7})$$

$$\lambda \frac{e_u^{k+1}}{\gamma (\sigma_{Q,k}^i)^2 \left(1 - (\rho_{Qq,k}^i)^2\right)} + (1 - \lambda) \frac{e_i^{k+1} \left(\mathbf{I}_{-2} - \frac{1}{p_{i2}^k} \mathbf{c}_2^T \mathbf{p}_i^k \mathbf{I}_{-2}\right) + e_u^{k+1}}{\gamma (\sigma_{Q,k}^u)^2} = 0. \quad (\text{A.8})$$

The first set of equations gives, for all k ,

$$e_0^{k+1} = \frac{\gamma (\sigma_{Q,k}^i)^2 \left(1 - (\rho_{Qq,k}^i)^2\right) (\sigma_{Q,k}^u)^2}{\lambda (\sigma_{Q,k}^u)^2 + (1 - \lambda) (\sigma_{Q,k}^i)^2 \left(1 - (\rho_{Qq,k}^i)^2\right)} > 0. \quad (\text{A.9})$$

Using

$$\mathbf{e}_i^{k+1} \left(\mathbf{c}_2^T \mathbf{c}_2 + \frac{1}{p_{i2}^k} \mathbf{c}_2^T \mathbf{p}_i^k \mathbf{I}_{-2}\right) = e_{i2}^{k+1} \begin{bmatrix} \frac{p_{i1}^k}{p_{i2}^k} & 1 & \frac{p_{i3}^k}{p_{i2}^k} & \dots & \frac{p_{i,K+3}^k}{p_{i2}^k} \end{bmatrix},$$

in (A.7), gives

$$\begin{aligned} & \frac{\lambda}{\gamma} \frac{e_{i1}^{k+1}}{(\sigma_{Q,k}^i)^2 \left(1 - (\rho_{Qq,k}^i)^2\right)} + \frac{1 - \lambda}{\gamma} \frac{e_{i2}^{k+1} \frac{p_{i1}^k}{p_{i2}^k}}{(\sigma_{Q,k}^u)^2} = 0, \\ & \frac{\lambda}{\gamma} \left[\frac{e_{i2}^{k+1}}{(\sigma_{Q,k}^i)^2 \left(1 - (\rho_{Qq,k}^i)^2\right)} - \frac{\rho_{Qq,k}^i}{\sigma_{Q,k}^i \sigma_{q,k}^i \left(1 - (\rho_{Qq,k}^i)^2\right)} \right] + \frac{1 - \lambda}{\gamma} \frac{e_{i2}^{k+1}}{(\sigma_{Q,k}^u)^2} = 0, \\ & \frac{\lambda}{\gamma} \frac{e_{i3}^{k+1}}{(\sigma_{Q,k}^i)^2 \left(1 - (\rho_{Qq,k}^i)^2\right)} + \frac{1 - \lambda}{\gamma} \frac{e_{i2}^{k+1} \frac{p_{i3}^k}{p_{i2}^k}}{(\sigma_{Q,k}^u)^2} = 0, \\ & \dots, \end{aligned}$$

which can be solved for

$$e_{i1}^{k+1} = -\frac{p_{i1}^k}{p_{i2}^k} \frac{(1-\lambda)(\sigma_Q^i)^2(1-(\rho_{Qq}^i)^2)}{\lambda(\sigma_Q^u)^2 + (1-\lambda)(\sigma_Q^i)^2(1-(\rho_{Qq}^i)^2)} \frac{Cov_t^i(Q_{t+1}, q_{t+1})}{(\sigma_q^i)^2}, \quad (\text{A.10})$$

$$e_{i2}^{k+1} = \frac{\lambda(\sigma_Q^u)^2}{\lambda(\sigma_Q^u)^2 + (1-\lambda)(\sigma_Q^i)^2(1-(\rho_{Qq}^i)^2)} \frac{Cov_t^i(Q_{t+1}, q_{t+1})}{(\sigma_q^i)^2}, \quad (\text{A.11})$$

$$e_{i3}^{k+1} = \frac{p_{i3}^k}{p_{i1}^k} e_{i1}^{k+1},$$

...

Turning now to (A.8), use

$$\mathbf{e}_i^{k+1} \left(\mathbf{I}_{-2} - \frac{1}{p_{i2}^k} \mathbf{c}_2 \mathbf{I}_i^k \mathbf{I}_{-2} \right) = \begin{bmatrix} e_{i1}^{k+1} - e_{i2}^{k+1} \frac{p_{i1}^k}{p_{i2}^k} & 0 & e_{i3}^{k+1} - e_{i2}^{k+1} \frac{p_{i3}^k}{p_{i2}^k} & \dots \end{bmatrix},$$

to derive

$$\begin{aligned} 0 &= \frac{\lambda}{\gamma} \frac{e_{u1}^{k+1}}{(\sigma_Q^i)^2(1-\rho_{Qq}^2)} + (1-\lambda) \frac{e_{i1}^{k+1} - e_{i2}^{k+1} \frac{p_{i1}^k}{p_{i2}^k} + e_{u1}^{k+1}}{\gamma(\sigma_Q^u)^2}, \\ 0 &= \frac{\lambda}{\gamma} \frac{e_{u2}^{k+1}}{(\sigma_Q^i)^2(1-\rho_{Qq}^2)} + (1-\lambda) \frac{e_{u2}^{k+1}}{\gamma(\sigma_Q^u)^2}, \\ 0 &= \frac{\lambda}{\gamma} \frac{e_{u3}^{k+1}}{(\sigma_Q^i)^2(1-\rho_{Qq}^2)} + (1-\lambda) \frac{e_{i3}^{k+1} - e_{i2}^{k+1} \frac{p_{i3}^k}{p_{i2}^k} + e_{u3}^{k+1}}{\gamma(\sigma_Q^u)^2}, \\ &\dots \end{aligned}$$

Solving these equations yields

$$\begin{aligned} e_{u1}^{k+1} &= \frac{p_{i1}^k}{p_{i2}^k} \frac{(1-\lambda)(\sigma_Q^i)^2(1-\rho_{Qq}^2)}{\lambda(\sigma_Q^u)^2 + (1-\lambda)(\sigma_Q^i)^2(1-\rho_{Qq}^2)} \frac{Cov_t^i(Q_{t+1}, q_{t+1})}{(\sigma_q^i)^2}, \quad (\text{A.12}) \\ e_{u2}^{k+1} &= 0, \\ e_{u3}^{k+1} &= \frac{p_{i3}^k}{p_{i1}^k} e_{u1}^{k+1}, \\ &\dots \end{aligned}$$

The equilibrium is a solution to $2(K+3)(K+1)$ price coefficients p^k , \mathbf{p}_i^k , and \mathbf{p}_u^k . Note that $p_{u2}^k = 0$. Equations (A.3) and (A.9) can be combined to yield $K+1$ equations that

can be solved for p^k , as will be done below. Combining equations (A.4) and (A.10) yields $(K+1) \times (K+3)$ equations and combining equations (A.5) and (A.12) yields the remaining $(K+1) \times (K+2)$ equations. Note that the equations for e_{u2}^k are redundant. When solving these equations, I substitute for the values of the conditional variances and covariances of returns computed above and that depend on the filtering problem of uninformed investors. A solution to this nonlinear system of equations constitutes a stationary rational expectations equilibrium.

It is possible to further characterize the equilibrium solution. Note that $e_{il}^{k+1} + e_{ul}^{k+1} = 0$ for all l but for $l = 2$. Thus, provided $t+1$ is not a dividend paying period (i.e., $K-1 \geq k \geq 0$), adding (A.4) and (A.5) gives

$$\mathbf{e}_i^{k+1} + \mathbf{e}_u^{k+1} = \mathbf{p}_i^{k+1} \mathbf{A}_x - R \mathbf{p}_i^k + \mathbf{p}_u^{k+1} \mathbf{I}_{-2} \mathbf{A}_x - R \mathbf{p}_u^k \mathbf{I}_{-2},$$

which can be simplified to yield

$$0 = (p_{i1}^{k+1} + p_{u1}^{k+1}) \rho_F - R (p_{i1}^k + p_{u1}^k) \quad (\text{A.13})$$

$$e_{i2}^{k+1} = p_{i2}^{k+1} \rho_Z - R p_{i2}^k \quad (\text{A.14})$$

$$\dots \quad (\text{A.15})$$

$$0 = p_{iK+3}^{k+1} + p_{uK+3}^{k+1} - R (p_{iK+2}^k + p_{uK+2}^k)$$

$$0 = p_{iK+3}^k + p_{uK+3}^k.$$

If $t+1$ is a dividend paying period (i.e., $K = k$), one obtains

$$\mathbf{e}_i^0 + \mathbf{e}_u^0 = (\mathbf{p}_i^0 + \mathbf{c}_{-2}) \mathbf{A}_x - R \mathbf{p}_i^K + \mathbf{p}_u^0 \mathbf{I}_{-2} \mathbf{A}_x - R \mathbf{p}_u^K \mathbf{I}_{-2},$$

which can be simplified to yield $0 = p_{u2}^k$, and

$$0 = (p_{i1}^0 + 1 + p_{u1}^0) \rho_F - R (p_{i1}^K + p_{u1}^K) \quad (\text{A.16})$$

$$e_{i2}^0 = p_{i2}^0 \rho_Z - R p_{i2}^K \quad (\text{A.17})$$

$$\dots \quad (\text{A.18})$$

$$0 = p_{iK+3}^0 + 1 + p_{uK+3}^0 - R (p_{iK+2}^K + p_{uK+2}^K)$$

$$0 = p_{iK+3}^K + p_{uK+3}^K.$$

Putting it all together gives the following solution to the coefficients of the price function.

First, I get the solution for $\{p^k\}$ from (A.3) and (A.9):

$$\begin{bmatrix} p^0 \\ p^1 \\ \dots \\ p^K \end{bmatrix} = \frac{-1}{R^{K+1} - 1} \begin{bmatrix} R^K & R^{K-1} & \dots & 1 \\ 1 & R^K & \dots & R \\ \dots & \dots & \dots & \dots \\ R^{K-1} & \dots & 1 & R^K \end{bmatrix} \begin{bmatrix} e_0^1 \\ e_0^2 \\ \dots \\ e_0^0 \end{bmatrix},$$

where each $p^k < 0$. Next, I get the solution for $\{p_{i2}^k\}$ by combining (A.11) with (A.14) and (A.17):

$$\begin{bmatrix} p_{i2}^0 \\ \dots \\ p_{i2}^{K-1} \\ p_{i2}^K \end{bmatrix} = \frac{-1}{R^{K+1} - \rho_F^{K+1}} \begin{bmatrix} \rho_F^K & R^K & R^{K-1}\rho_F & \dots & R\rho_F^{K-1} \\ R\rho_F^{K-1} & \rho_F^K & R^K & \dots & R^2\rho_F^{K-2} \\ \dots & \dots & \dots & \dots & \dots \\ R^{K-1}\rho_F & R^{K-2}\rho_F^2 & R^{K-3}\rho_F^3 & \dots & R^K \\ R^K & R^{K-1}\rho_F & R^{K-2}\rho_F^2 & \dots & \rho_F^K \end{bmatrix} \begin{bmatrix} e_{i2}^0 \\ \dots \\ e_{i2}^{K-1} \\ e_{i2}^K \end{bmatrix}. \quad (\text{A.19})$$

Note that if $Cov_{Qq,k}^i > 0$, for all k , then $p_{i2}^k < 0$, for all k .

Further, it is possible to solve for $p_{i1}^k + p_{u1}^k$ using (A.13) and (A.16):

$$p_{i1}^k + p_{u1}^k = \frac{R^k \rho_F^{K+1-k}}{R^{K+1} - \rho_F^{K+1}}.$$

Collecting the remaining equation in (A.15) and (A.18), I get $p_{iK+3}^k + p_{uK+3}^k = 0$ for all k , and

$$\begin{aligned} p_{iK+2}^K + p_{uK+2}^K &= R^{-1} \\ p_{iK+2}^k + p_{uK+2}^k &= 0, \quad k = 0, \dots, K-1 \end{aligned}$$

$$\begin{aligned} p_{iK+1}^K + p_{uK+1}^K &= R^{-1} \\ p_{iK+1}^{K-1} + p_{uK+1}^{K-1} &= R^{-2} \\ p_{iK+1}^k + p_{uK+1}^k &= 0, \quad k = 0, \dots, K-2 \end{aligned}$$

...

$$\begin{aligned} p_{i3}^K + p_{u3}^K &= R^{-1} \\ p_{i3}^{K-1} + p_{u3}^{K-1} &= R^{-2} \end{aligned}$$

...

$$\begin{aligned} p_{i3}^1 + p_{u3}^1 &= R^{-K} \\ p_{i3}^k + p_{u3}^k &= 0, \quad k = 0. \end{aligned}$$

To complete the proof, I now show that

$$\begin{aligned}
p_{iK+3}^K &= p_{uK+3}^K = 0 \\
p_{iK+2}^{K-1} &= p_{uK+2}^{K-1} = p_{iK+3}^{K-1} = p_{uK+3}^{K-1} = 0 \\
&\dots \\
p_{i4}^1 &= p_{u4}^1 = \dots = p_{iK+2}^1 = p_{uK+2}^1 = p_{iK+3}^1 = p_{uK+3}^1 = 0 \\
p_{i3}^0 &= p_{u3}^0 = p_{i4}^0 = p_{u4}^0 = \dots = p_{iK+2}^0 = p_{uK+2}^0 = p_{iK+3}^0 = p_{uK+3}^0 = 0.
\end{aligned}$$

I start with showing that $p_{iK+3}^k = p_{uK+3}^k = 0$. Start with (A.5) and assume that $k+1 \neq 0$, or $k < K$. It can be shown that

$$\mathbf{e}_u^{k+1} = \begin{bmatrix} p_{u1}^{k+1} \rho_F - R p_{u1}^k \\ 0 \\ p_{u4}^{k+1} - R p_{u3}^k \\ \dots \\ p_{uK+3}^{k+1} - R p_{uK+2}^k \\ -R p_{uK+3}^k \end{bmatrix}^\top - \mathbf{p}_u^{k+1} \mathbf{K}_{k+1}^k \times \quad (\text{A.20})$$

$$\begin{bmatrix} p_{i1}^{k+1} \rho_F - p_{i2}^{k+1} \frac{p_{i1}^k}{p_{i2}^k} \rho_Z & 0 & p_{i4}^{k+1} - p_{i2}^{k+1} \frac{p_{i3}^k}{p_{i2}^k} \rho_Z & \dots & -p_{i2}^{k+1} \frac{p_{iK+3}^k}{p_{i2}^k} \rho_Z \\ \rho_F & 0 & \dots & \dots & 0 \\ 0 & 0 & 0 & \dots & 0 \end{bmatrix}, (\text{A.21})$$

with the term on the right hand side post-multiplying \mathbf{p}_u^{k+1} equal to

$$\begin{bmatrix} k_{11}^k \left(p_{i1}^{k+1} \rho_F - p_{i2}^{k+1} \frac{p_{i1}^k}{p_{i2}^k} \rho_Z \right) + k_{12}^k \rho_F & 0 & k_{11}^k \left(p_{i4}^{k+1} - p_{i2}^{k+1} \frac{p_{i3}^k}{p_{i2}^k} \rho_Z \right) & \dots & -p_{i2}^{k+1} \frac{p_{iK+3}^k}{p_{i2}^k} \rho_Z k_{11}^k \\ k_{21}^k \left(p_{i1}^{k+1} \rho_F - p_{i2}^{k+1} \frac{p_{i1}^k}{p_{i2}^k} \rho_Z \right) + k_{22}^k \rho_F & 0 & k_{21}^k \left(p_{i4}^{k+1} - p_{i2}^{k+1} \frac{p_{i3}^k}{p_{i2}^k} \rho_Z \right) & & -p_{i2}^{k+1} \frac{p_{iK+3}^k}{p_{i2}^k} \rho_Z k_{21}^k \\ \dots & & & & \\ \dots & & & & \\ 0 & & & & -p_{i2}^{k+1} \frac{p_{iK+3}^k}{p_{i2}^k} \rho_Z k_{K+31}^k \end{bmatrix}.$$

Then, using the last row of (A.20) for any $k < K$, and letting $\mathbf{K}_{k+1,1}^k$ denote the first column of \mathbf{K}_{k+1}^k ,

$$e_{uK+3}^{k+1} = -R p_{uK+3}^k + p_{i2}^{k+1} \frac{p_{iK+3}^k}{p_{i2}^k} \rho_Z \mathbf{p}_u^{k+1} \mathbf{K}_{k+1,1}^k.$$

Replacing the value of e_{uK+3}^{k+1} by its expression in (A.12) and using the fact that $p_{uK+3}^k + p_{iK+3}^k = 0$, the previous equality can be written as

$$\frac{p_{iK+3}^k}{p_{i2}^k} \frac{(1-\lambda) (\sigma_Q^i)^2 (1-\rho_{Qq}^2)}{\lambda (\sigma_Q^u)^2 + (1-\lambda) (\sigma_Q^i)^2 (1-\rho_{Qq}^2)} \frac{\text{Cov}_t^i(Q_{t+1}, q_{t+1})}{(\sigma_q^i)^2} = R p_{iK+3}^k + p_{i2}^{k+1} \frac{p_{iK+3}^k}{p_{i2}^k} \rho_Z \mathbf{p}_u^{k+1} \mathbf{K}_{k+1,1}^k.$$

Suppose $p_{iK+3}^k \neq 0$, then

$$\frac{1}{p_{i2}^k} \frac{(1-\lambda) \left(\sigma_Q^i\right)^2 \left(1-\rho_{Qq}^2\right)}{\lambda \left(\sigma_Q^u\right)^2 + (1-\lambda) \left(\sigma_Q^i\right)^2 \left(1-\rho_{Qq}^2\right)} \frac{Cov_t^i(Q_{t+1}, q_{t+1})}{\left(\sigma_q^i\right)^2} = R + \frac{p_{i2}^{k+1}}{p_{i2}^k} \rho_Z \mathbf{p}_u^{k+1} \mathbf{K}_{k+1,1}^k. \quad (\text{A.22})$$

Now note that for $k < K$ ($k+1 \neq 0$), the next to last equation in (A.20) is

$$e_{uK+2}^{k+1} = p_{uK+3}^{k+1} - R p_{uK+2}^k - \mathbf{p}_u^{k+1} \mathbf{K}_{k+1,1}^k \left(p_{iK+3}^{k+1} - p_{i2}^{k+1} \frac{p_{iK+2}^k}{p_{i2}^k} \rho_Z \right),$$

or

$$\begin{aligned} & \frac{p_{iK+2}^k}{p_{i2}^k} \frac{(1-\lambda) \left(\sigma_Q^i\right)^2 \left(1-\rho_{Qq}^2\right)}{\lambda \left(\sigma_Q^u\right)^2 + (1-\lambda) \left(\sigma_Q^i\right)^2 \left(1-\rho_{Qq}^2\right)} \frac{Cov_t^i(Q_{t+1}, q_{t+1})}{\left(\sigma_q^i\right)^2} - R p_{iK+2}^k \\ &= \mathbf{p}_u^{k+1} \mathbf{K}_{k+1,1}^k \frac{p_{iK+2}^k}{p_{i2}^k} \rho_Z - p_{iK+3}^{k+1} \left(1 + \mathbf{p}_u^{k+1} \mathbf{K}_{k+1,1}^k \right). \end{aligned}$$

Again, replacing the value of e_{uK+2}^{k+1} and using (A.22), I arrive at

$$0 = -p_{iK+3}^{k+1} \left(1 + \mathbf{p}_u^{k+1} \mathbf{K}_{k+1,1}^k \right).$$

Under the assumption that $p_{iK+3}^{k+1} \neq 0$, it must then be that $1 + \mathbf{p}_u^{k+1} \mathbf{K}_{k+1,1}^k = 0$. But, from (A.14),

$$e_{i2}^{k+1} = p_{i2}^{k+1} \rho_Z - R p_{i2}^k,$$

which combined with (A.22) and the equilibrium value of e_{i2}^{k+1} , leads to

$$\frac{Cov_t^i(Q_{t+1}, q_{t+1})}{\left(\sigma_q^i\right)^2} = p_{i2}^{k+1} \rho_Z \left(1 + \mathbf{p}_u^{k+1} \mathbf{K}_{k+1,1}^k \right).$$

This results in a contradiction unless the two assets are uncorrelated, i.e., $Cov_t^i(Q_{t+1}, q_{t+1}) = 0$. Next, I show that $p_{iK+2}^{K-1} = p_{uK+2}^{K-1} = 0$. (The proof for the other price coefficients is similar.)

From the next to last equation in (A.20) written for $k = K - 1$:

$$\begin{aligned} p_{uK+3}^K - R p_{uK+2}^{K-1} - \mathbf{p}_u^K \mathbf{K}_{K,1}^{K-1} \left(p_{iK+3}^K - p_{i2}^K \frac{p_{iK+2}^{K-1}}{p_{i2}^{K-1}} \rho_Z \right) &= e_{uK+2}^K \\ -R p_{uK+2}^{K-1} + \mathbf{p}_u^K \mathbf{K}_{K,1}^{K-1} p_{i2}^K \frac{p_{iK+2}^{K-1}}{p_{i2}^{K-1}} \rho_Z &= e_{uK+2}^K. \end{aligned}$$

Using the expression for e_{uK+2}^K , $p_{iK+2}^{K-1} + p_{uK+2}^{K-1} = 0$, and letting $p_{iK+2}^{K-1} \neq 0$, then the last expression can be written as

$$R + \mathbf{p}_u^K \mathbf{K}_{K,1}^{K-1} \frac{p_{i2}^K}{p_{i2}^{K-1}} \rho_Z = \frac{1}{p_{i2}^{K-1}} \frac{(1-\lambda) \left(\sigma_Q^i\right)^2 \left(1-\rho_{Qq}^2\right)}{\lambda \left(\sigma_Q^u\right)^2 + (1-\lambda) \left(\sigma_Q^i\right)^2 \left(1-\rho_{Qq}^2\right)} \frac{Cov_t^i(Q_{t+1}, q_{t+1})}{\left(\sigma_q^i\right)^2},$$

which it was just shown to not hold. The contradiction implies $p_{iK+2}^{K-1} = p_{uK+2}^{K-1} = 0$. ■

Proof of Proposition 3: Throughout, I shall use the result proved above that $e_{il}^{k+1} + e_{ul}^{k+1} = 0$ for all l but for $l = 2$. Using (10) and the expression for $E_t^u [Q_{t+1}^{k+1}]$, the stock demand of uninformed investors is

$$\theta_t^u = \frac{e_0^{k+1} + (\mathbf{e}_i^{k+1} + \mathbf{e}_u^{k+1}) \hat{\mathbf{x}}_t}{\gamma (\sigma_Q^u)^2} = \frac{e_0^{k+1} + e_{i2}^{k+1} \hat{Z}_t}{\gamma (\sigma_Q^u)^2}.$$

The asset demand of informed investors can be similarly obtained. Using (9) and the expression for $E_t^i [Q_{t+1}^{k+1}]$, I get

$$\begin{aligned} \theta_t^i &= \frac{e_0^{k+1} + e_{i2}^{k+1} Z_t - \mathbf{e}_u^{k+1} (\mathbf{x}_t - \hat{\mathbf{x}}_t)}{\gamma (\sigma_Q^i)^2 (1 - (\rho_{Qq}^i)^2)} - \frac{\rho_{Qq}^i Z_t}{\gamma \sigma_Q^i \sigma_q^i (1 - (\rho_{Qq}^i)^2)} \\ &= \frac{e_0^{k+1}}{\gamma (\sigma_Q^i)^2 (1 - (\rho_{Qq}^i)^2)} - \left(\frac{\rho_{Qq}^i}{\gamma \sigma_Q^i \sigma_q^i (1 - (\rho_{Qq}^i)^2)} - \frac{e_{i2}^{k+1}}{\gamma (\sigma_Q^i)^2 (1 - (\rho_{Qq}^i)^2)} \right) Z_t \\ &\quad - \frac{e_{u1}^{k+1}}{\gamma (\sigma_Q^i)^2 (1 - (\rho_{Qq}^i)^2)} (F_t - \hat{F}_t) - \sum_{j=0}^{k-1} \frac{e_{uj+3}^{k+1}}{\gamma (\sigma_Q^i)^2 (1 - (\rho_{Qq}^i)^2)} (\varepsilon_{t-j}^D - \hat{\varepsilon}_{t-j}^D). \end{aligned}$$

Using the expression for e_{i2}^{k+1} in equation (A.11), the coefficient associated with Z_t can be written as

$$-\frac{\rho_{Qq}^i}{\gamma \sigma_Q^i \sigma_q^i (1 - (\rho_{Qq}^i)^2)} \frac{(1 - \lambda) (\sigma_Q^i)^2 (1 - (\rho_{Qq}^i)^2)}{\lambda (\sigma_Q^u)^2 + (1 - \lambda) (\sigma_Q^i)^2 (1 - (\rho_{Qq}^i)^2)},$$

which is negative if and only if $Cov_t^i(Q_{t+1}, q_{t+1}) > 0$. Summarizing, because $e_0^{k+1} > 0$, $f_{i0}^{k+1}, f_{u0}^{k+1} > 0$. Also, $f_{i1}^{k+1} < 0$ and $f_{u1}^{k+1} > 0$ if and only if $Cov_t^i(Q_{t+1}, q_{t+1}) > 0$. ■

Conditional volatility of returns and mean trading volume calculations: Compute the value of the stock return variance conditional only on knowing that returns are drawn

from a period of type k :

$$\begin{aligned}
\sigma_{Q,k}^2 &= E_k \left[\left(Q_{t+1}^{k+1} - e_0^{k+1} \right)^2 \right] \\
&= E_k \left[\left(\mathbf{e}_i^{k+1} \mathbf{x}_t + \mathbf{e}_u^{k+1} \hat{\mathbf{x}}_t + \mathbf{b}_Q^{k+1} \boldsymbol{\varepsilon}_{t+1}^{k+1} \right)^2 \right] \\
&= E_k \left[E_t^u \left(\mathbf{e}_i^{k+1} (\mathbf{x}_t - \hat{\mathbf{x}}_t) + e_{i2}^{k+1} \hat{Z}_t + \mathbf{b}_Q^{k+1} \boldsymbol{\varepsilon}_{t+1}^{k+1} \right)^2 \right] \\
&= \mathbf{b}_Q^{k+1} \Sigma_{\varepsilon\varepsilon}^{k+1} \left(\mathbf{b}_Q^{k+1} \right)^\top + \left(e_{i2}^{k+1} \right)^2 E_k \left[\hat{Z}_t^2 \right] + \mathbf{e}_i^{k+1} \Omega^k \left(\mathbf{e}_i^{k+1} \right)^\top,
\end{aligned}$$

recalling that $E_k \left[(\mathbf{x}_t - \hat{\mathbf{x}}_t) \hat{Z}_t \right] = E_k \left[E_t^u \left((\mathbf{x}_t - \hat{\mathbf{x}}_t) \hat{Z}_t \right) \right] = E_k \left[E_t^u (\mathbf{x}_t - \hat{\mathbf{x}}_t) \hat{Z}_t \right] = 0$, using the law of iterated expectations. To compute $E_k \left[\hat{Z}_t^2 \right]$, note that, by definition of Ω^k ,

$$\hat{\mathbf{x}}_t \hat{\mathbf{x}}_t^\top = E_t^u [\mathbf{x}_t \mathbf{x}_t^\top] - \Omega^k.$$

Thus, $E_k [\hat{\mathbf{x}}_t \hat{\mathbf{x}}_t^\top] = E_k [\mathbf{x}_t \mathbf{x}_t^\top] - \Omega^k$. Turn now to the value of $E_k [\mathbf{x}_t \mathbf{x}_t^\top]$. First, note that conditional on (t, k) , then $t-1$ is a $k-1$ -period and, abusing notation slightly, $E_k [\mathbf{x}_{t-1} \mathbf{x}_{t-1}^\top] = E_{k-1} [\mathbf{x}_{t-1} \mathbf{x}_{t-1}^\top]$. Then,

$$\begin{aligned}
E_k [\mathbf{x}_t \mathbf{x}_t^\top] &= E_k \left[E_{t-1}^i (\mathbf{x}_t \mathbf{x}_t^\top) \right] \\
&= \mathbf{A}_x E_{k-1} [\mathbf{x}_{t-1} \mathbf{x}_{t-1}^\top] \mathbf{A}_x^\top + \mathbf{B}_x \Sigma_{\varepsilon\varepsilon}^k \mathbf{B}_x^\top
\end{aligned}$$

which can be found by solving the system of $K+1$ equations on $E_k [\mathbf{x}_t \mathbf{x}_t^\top]$ and noting that

$$E_0 [\mathbf{x}_t \mathbf{x}_t^\top] = \mathbf{A}_x E_K [\mathbf{x}_{t-1} \mathbf{x}_{t-1}^\top] \mathbf{A}_x^\top + \mathbf{B}_x \Sigma_{\varepsilon\varepsilon}^0 \mathbf{B}_x^\top.$$

The value of $E_k \left[\hat{Z}_t^2 \right]$ is the $(2, 2)$ element of $E_k [\mathbf{x}_t \mathbf{x}_t^\top] - \Omega^k$.

To compute the mean trading volume, I solve for $\sigma_{\Delta\theta_t^u, k}^2$,

$$\begin{aligned}
\sigma_{\Delta\theta_t^u, k}^2 &= E_k \left[\left(\theta_t^u - \theta_{t-1}^u - E_k (\theta_t^u - \theta_{t-1}^u) \right)^2 \right] \\
&= E_k \left[\left(f_{u1}^{k+1} \hat{Z}_t - f_{u1}^k \hat{Z}_{t-1} \right)^2 \right] \\
&= \left(f_{u1}^{k+1} \right)^2 E_k \left[\hat{Z}_t^2 \right] + \left(f_{u1}^k \right)^2 E_{k-1} \left(\hat{Z}_{t-1}^2 \right) - 2 f_{u1}^{k+1} f_{u1}^k E_k \left(\hat{Z}_t \hat{Z}_{t-1} \right) \\
&= \left(f_{u1}^{k+1} \right)^2 E_k \left[\hat{Z}_t^2 \right] + \left(f_{u1}^k - 2 f_{u1}^{k+1} \rho_Z \right) f_{u1}^k E_{k-1} \left(\hat{Z}_{t-1}^2 \right),
\end{aligned}$$

where the third equality uses the fact that conditioning on t being a k -type period then $t-1$ is a $k-1$ -type period and $E_k \left(\hat{Z}_{t-1}^2 \right) = E_{k-1} \left(\hat{Z}_{t-1}^2 \right)$. The last equality uses the fact that the conditional error in the expression for \hat{Z}_t is independent of \hat{Z}_{t-1} . ■

Proof of Corollary 1: Using the definition of $f(Q)$, the unconditional mean stock return is

$$E(Q_{t+1}) = \frac{1}{K+1} \sum_{k=0}^K E_k(Q_{t+1}) = \frac{1}{K+1} \sum_{k=0}^K e_0^k.$$

The unconditional variance in stock returns is

$$\begin{aligned} Var(Q_{t+1}) &= \frac{1}{K+1} \sum_{k=0}^K \int (Q - E(Q_{t+1}))^2 \phi(Q; e_0^k, \sigma_{Q,k}^2) dQ \\ &= \frac{1}{K+1} \sum_{k=0}^K \int (Q - e_0^k + e_0^k - E(Q_{t+1}))^2 \phi(Q; e_0^k, \sigma_{Q,k}^2) dQ \\ &= \frac{1}{K+1} \sum_{k=0}^K \left(\sigma_{Q,k}^2 + (e_0^k - E(Q_{t+1}))^2 \right). \end{aligned}$$

Finally, unconditional skewness is

$$\begin{aligned} E[(Q - E(Q_{t+1}))^3] &= \frac{1}{K+1} \sum_{k=0}^K \int (Q - E(Q_{t+1}))^3 \phi(Q; e_0^k, \sigma_{Q,k}^2) dQ \\ &= \frac{1}{K+1} \sum_{k=0}^K \int (Q - e_0^k + e_0^k - E(Q_{t+1}))^3 \phi(Q; e_0^k, \sigma_{Q,k}^2) dQ \\ &= \frac{1}{K+1} \sum_{k=0}^K \left[(e_0^k - E(Q_{t+1}))^3 + 3\sigma_{Q,k}^2 (e_0^k - E(Q_{t+1})) \right]. \end{aligned}$$

The second term under the summation sign can be manipulated to yield the expression in the corollary by noting that

$$e_0^k - E(Q_{t+1}) = \frac{1}{K+1} \sum_{j=0, j \neq k}^K (e_0^k - e_0^j),$$

and grouping terms together under the last summation sign. ■

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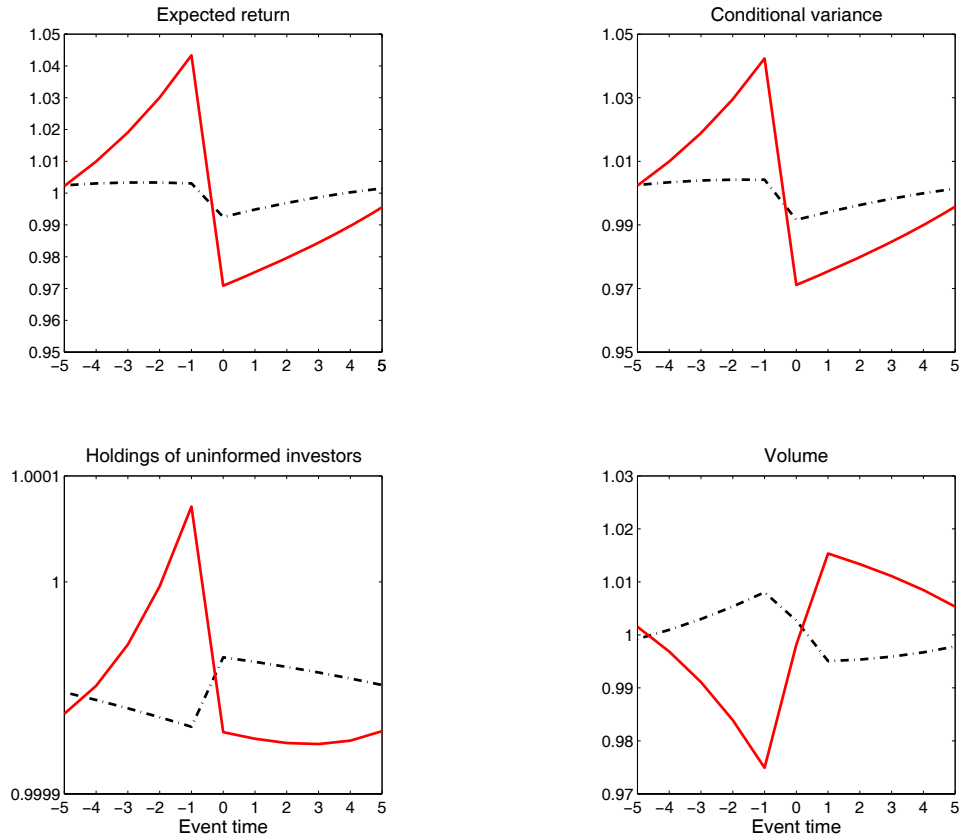


Figure 1: **Model without asymmetric information.** Pictures depict the event window around the dividend announcement, which occurs at date 0. Expected return is $E_k [Q_{t+1}^{k+1}]$, conditional volatility is $\sigma_{Q,k}^2$, holdings of uninformed investors is $(1 - \lambda) E_k [\theta_t^u]$ and conditional trading volume is $E_k [Vol_t]$. Variables have been normalized to have mean of one. The solid line is for $\sigma_Z^2 = 2$ and the dashed line is for $\sigma_Z^2 = 5$. Public signals are fully informative, $\sigma_{Dk}^2 = \sigma_{Fk}^2 = 0$. Remaining parameters are: $K = 10$, $\sigma_D^2 = \sigma_q^2 = \sigma_F^2 = 1$, $\sigma_{Dq} = .5$, $\rho_F = \rho_Z = .9$, $\gamma = 5$, $\lambda = .5$, $R = 1.0025$.

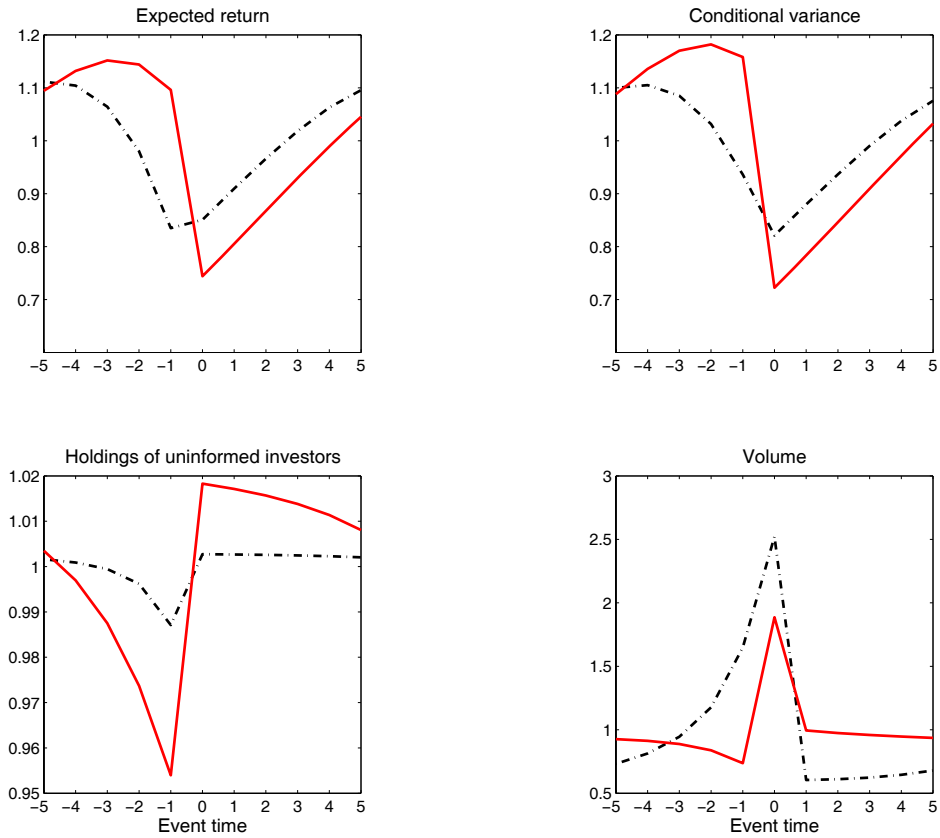


Figure 2: **Model with asymmetric information.** Pictures depict the event window around the dividend announcement, which occurs at date 0. Expected return is $E_k [Q_{t+1}^{k+1}]$, conditional volatility is $\sigma_{Q,k}^2$, holdings of uninformed investors is $(1 - \lambda) E_k [\theta_t^u]$ and conditional trading volume is $E_k [Vol_t]$. Variables have been normalized to have mean of one. The solid line is for $\sigma_Z^2 = 1$ and the dashed line is for $\sigma_Z^2 = 5$. Public signals are not informative, $\sigma_{Dk}^2, \sigma_{Fk}^2 \rightarrow \infty$. Remaining parameters are: $K = 10$, $\sigma_D^2 = \sigma_q^2 = \sigma_F^2 = 1$, $\sigma_{Dq} = .5$, $\rho_F = \rho_Z = .9$, $\gamma = 5$, $\lambda = .5$, $R = 1.0025$.

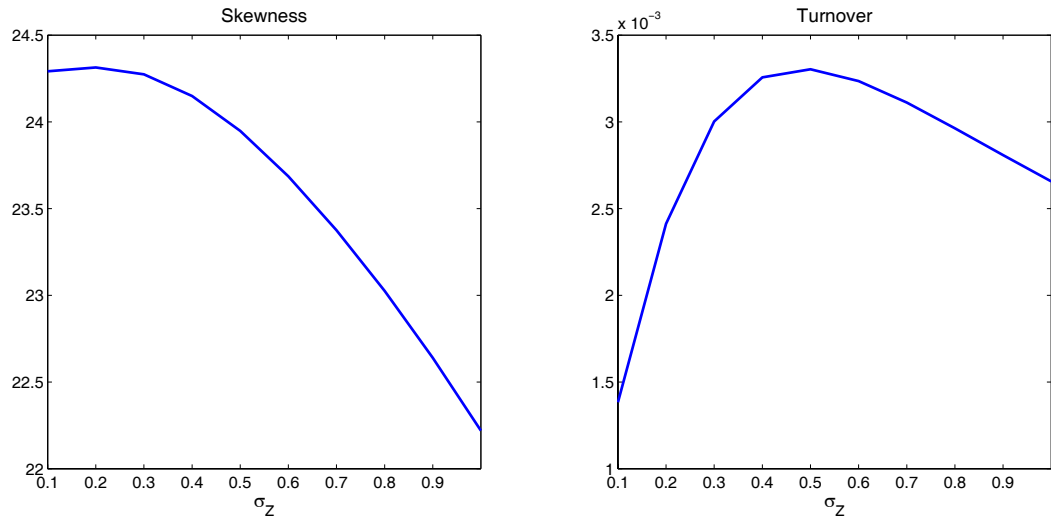


Figure 3: **Model without asymmetric information: The effect of liquidity shocks.** Public signals are fully informative, $\sigma_{Dk}^2 = \sigma_{Fk}^2 = 0$. The parameters used are: $K = 10$, $\sigma_D^2 = \sigma_q^2 = \sigma_F^2 = 1$, $\sigma_{Dq} = .5$, $\rho_F = \rho_Z = .9$, $\gamma = 5$, $\lambda = .5$, $R = 1.0025$.

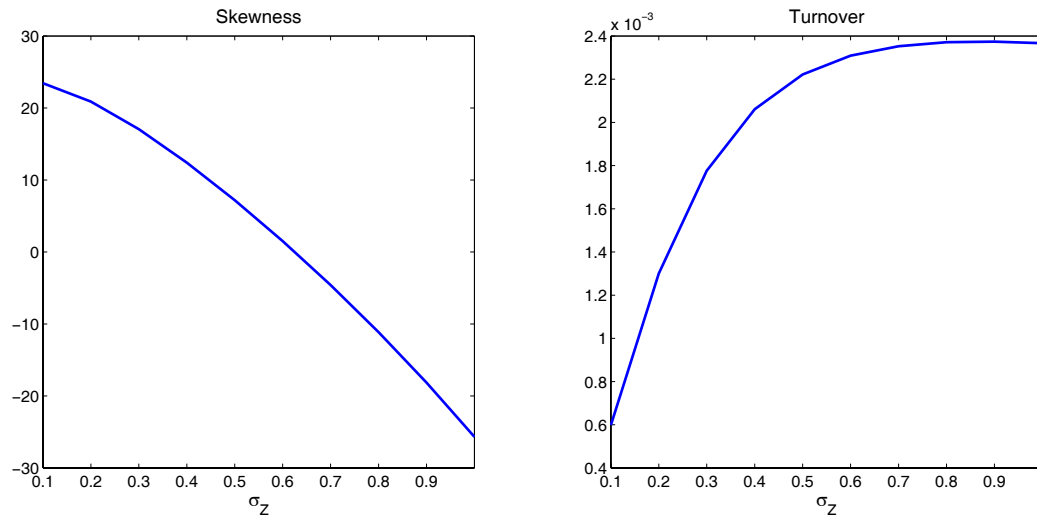


Figure 4: **Model with asymmetric information: The effect of liquidity shocks.** Public signals are not informative, $\sigma_{Dk}^2, \sigma_{Fk}^2 \rightarrow \infty$. The parameters used are: $K = 10$, $\sigma_D^2 = \sigma_q^2 = \sigma_F^2 = 1$, $\sigma_{Dq} = .5$, $\rho_F = \rho_Z = .9$, $\gamma = 5$, $\lambda = .5$, $R = 1.0025$.

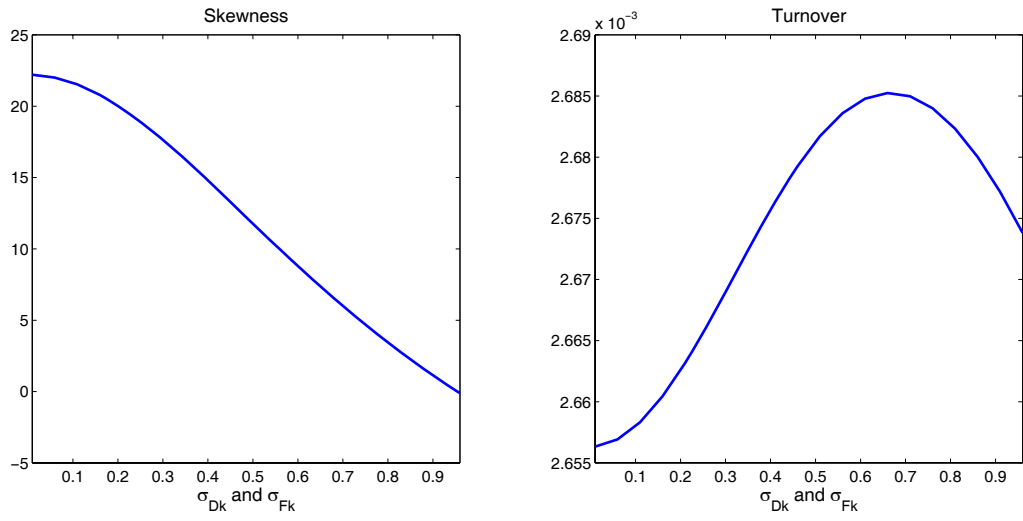


Figure 5: **Model with asymmetric information: The effect of information asymmetry.** The parameters used are: $K = 10$, $\sigma_Z^2 = \sigma_D^2 = \sigma_q^2 = \sigma_F^2 = 1$, $\sigma_{Dq} = .5$, $\rho_F = \rho_Z = .9$, $\gamma = 5$, $\lambda = .5$, $R = 1.0025$.