Gain = Volatility of Outcomes
      Volatility of Fundamentals

= +/-0

in SSE
Mea Culpa

- Totally unfair spoof of Jevons (1884)
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- Cass-Shell Sunspots ≡ Extrinsic Randomizing Device
Mea Culpa

- Totally unfair spoof of Jevons (1884)
- Cass-Shell Sunspots ≡ Extrinsic Randomizing Device
- Unfair to extrinsic uncertainty: too cute for central banks

Stanley Fischer
Roger Myerson
Exchange Economy

(P) \( V_h(x_h(\alpha), x(\beta)) = \pi(\alpha)u_h(x_h(\alpha)) + \pi(\beta)u_h(x_h(\beta)) \)

(E) \( \omega_h(\alpha) = \omega_h(\beta) = \omega_h > 0 \)

(BC) \( p(\alpha) \cdot x_h(\alpha) + p(\beta) \cdot x_h(\beta) = (p(\alpha) + p(\beta)) \cdot \omega_h \)

(CP) \( \max V_h \quad \text{s.t. BC} \)

(MB) \( \sum_h x_h(\alpha) = \sum_h \omega_h = \sum_h x_h(\beta) \)
Cass-Shell Sunspots Immunity Theorem

Perfect Markets in Finite, Competitive Sunspots Economy
Cass-Shell Sunspots Immunity Theorem

Perfect Markets in Finite, Competitive Sunspots Economy

Suppose $x_h(\alpha) \neq x_h(\beta)$ some $h$, i.e., sunspots matter
Cass-Shell Sunspots Immunity Theorem

Perfect Markets in Finite, Competitive Sunspots Economy

Suppose $x_h(\alpha) \neq x_h(\beta)$ some $h$, i.e., sunspots matter

$$\bar{x}_h = \pi(\alpha)x_h(\alpha) + \pi(\beta)x_h(\beta) \quad \text{all } h$$

$$V_h(\bar{x}_h, \bar{x}_h) = \pi(\alpha)u_h(\bar{x}_h) + \pi(\beta)u_h(\bar{x}_h)$$

$$= u_h(\pi(\alpha)x_h(\alpha) + \pi(\beta)x_h(\beta))$$

$$> \pi(\alpha)u_h(x_h(\alpha)) + \pi(\beta)u_h(x_h(\beta))$$

$$\sum_h \bar{x}_h = \pi(\alpha)\sum_h x_h(\alpha) + \pi(\beta)\sum_h x_h(\beta)$$

$$= (\pi(\alpha) + \pi(\beta))\sum_h \omega_h = \sum_h \omega_h$$

$\bar{x}_h$ is feasible and Pareto superior in sunspots economy.
Cass-Shell Sunspots Immunity Theorem

Perfect Markets in Finite, Competitive Sunspots Economy

Suppose $x_h(\alpha) \neq x_h(\beta)$ some $h$, i.e., sunspots matter

$$x_h = \pi(\alpha)x_h(\alpha) + \pi(\beta)x_h(\beta) \quad \text{all } h$$

$$V_h(\bar{x}_h, \bar{x}_h) = \pi(\alpha)u_h(\bar{x}_h) + \pi(\beta)u_h(\bar{x}_h)$$

$$= u_h(\pi(\alpha)x_h(\alpha) + \pi(\beta)x_h(\beta))$$

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$$= (\pi(\alpha) + \pi(\beta))\sum_h \omega_h = \sum_h \omega_h$$

$\bar{x}_h$ is feasible and Pareto superior in sunspots economy.

Subtle contradiction. Sunspots do not matter.

$$\sum_h \omega_h(\alpha) = \sum_h \omega_h(\beta)$$, hyper-cube Edgeworth box is key to proof.
Money (and Finance) are sources of Excess Volatility.

Information Frictions, the Tax-Adjusted Edgeworth Box, and the Source of SSE
3 consumers $h = 1, 2, 3$.

1 commodity

2 extrinsic states, $\alpha, \beta$. $\pi(\alpha) = 3/4, \pi(\beta) = 1/4$.

Mr 3 cannot see sunspots. Mr 1 & 2 can.

$\omega = (20, 10, 5)$

$\tau = (5, 0, -5), \sum_h \tau_h = 0$

$P^m(s)$ is chocolate price of money (inverse of price level)

$\alpha$ is the inflationary state $P^m(\alpha) = 1$, $P^m(\beta) = 2$ is the deflationary state

$x_3(\alpha) = \tilde{\omega}_3(\alpha) = 5 + 5 = 10$

$x_3(\beta) = \tilde{\omega}_3(\beta) = 5 + 10 = 15$
Mr 1 and Mr 2 trade in a proper rectangular 25 X 20 Edgeworth box defined by

\[ x_1(s) + x_2(s) = \tilde{\omega}_1(s) + \tilde{\omega}_2(s) \quad s = \alpha, \beta \]

where

\[ \tilde{\omega}_h(s) = \omega_h - P^m(s) \tau_h \quad h = 1, 2 \quad s = \alpha, \beta \]

\[ \max V_h = \pi(\alpha) \log x_h(\alpha) + \pi(\beta) \log x_h(\beta) \]

s.t

\[ \text{BC} \quad p(\alpha)x_h(\alpha) + p(\beta)x_h(\beta) = p(\alpha)\tilde{\omega}_h(\alpha) + p(\beta)\tilde{\omega}_h(\beta) \]
Figure: The $25 \times 20$ Tax-Adjusted Edgeworth Box
Solutions:

\[ (\tilde{\omega}_1 (\alpha), \tilde{\omega}_1 (\beta)) = (20 - 5, 20 - 10) = (15, 10) \]
\[ (\tilde{\omega}_2 (\alpha), \tilde{\omega}_2 (\beta)) = (10 - 0, 10 - 0) = (10, 10) \]
\[ (\tilde{\omega}_3 (\alpha), \tilde{\omega}_3 (\beta)) = (5 + 5, 5 + 10) = (10, 15) \]
\[ (x_3 (\alpha), x_3 (\beta)) = (10, 15) \]

\[ \tilde{\omega}_1 (\alpha) + \tilde{\omega}_2 (\alpha) = 15 + 10 = 25 = 20 + 10 + 5 - 10 \]
\[ \tilde{\omega}_1 (\beta) + \tilde{\omega}_2 (\beta) = 10 + 10 = 20 = 20 + 10 + 5 - 15 \]

\[ \frac{p(\alpha)}{p(\beta)} = \left( \frac{20}{25} \right) \left( \frac{3/4}{1/4} \right) = \frac{12}{5} \]

\[ x_1 (\alpha) = 14 \frac{3}{8}, \quad x_1 (\beta) = 11 \frac{1}{2} \]
\[ x_2 (\alpha) = 10 \frac{5}{8}, \quad x_2 (\beta) = 8 \frac{1}{2} \]
Sample Calculation 1: Money Taxation

- Mr. 1 is rich, $\omega_1 = 80$.
- Mr. 2 is middle-class, $\omega_2 = 60$, and holds the median = the mean endowment.
- Mr. 3 is poor, $\omega_3 = 40$, and also disadvantaged in trading.
- $\bar{\omega} = (80 + 60 + 40) / 3 = 60 = \omega_2 = \omega_{med}$.
Chicago Connections
Figure: “The Shelton with Sunspots”, Georgia O’Keefe, 1926.
Lucas
QTM
JPE
Fresh Water
REH
RBC
Diamond (& Dybvig)
Myerson
Stokey
Other Connections
- Cass
- Malinvaud
- Azariadis
- Balasko
- Peck
- Manuelli
- Wright
- Benhabib
- Farmer
- Salt Water
The Philadelphia Pholk “Theorem”
Overlapping Generations
- Overlapping Generations
- Restricted Participation
- Overlapping Generations
- Restricted Participation
- “Double Infinity”, Open horizon
- Overlapping Generations
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- Incomplete Financial Markets
- Bank Runs & Financial Fragility
- Non-Convexities
- Sunspots & Lotteries
- Imperfect competition and correlation (Information Friction)
- Empirical Applications
- Political Economy
- Winners and Losers from Volatility
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