Levelized Product Cost: Concept and Decision Relevance

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Abstract: Levelized Product Cost: Concept and Decision Relevance

This paper presents a cost concept that seeks to capture the unit cost of production when viewed over the entire life-cycle of a production facility. Borrowing from the energy literature, we refer to this life-cycle cost as the levelized cost of production (LPC). By construction, the LPC aggregates a share of the initial capacity costs with fixed and variable operating costs. We demonstrate that, given proper accrual accounting (depreciation rules), the levelized product cost can be aligned with the accountant’s notion of full cost. At the same time, the LPC can be interpreted as a long-run marginal cost which under certain conditions becomes the relevant cost for making optimal capacity investments. We demonstrate the decision relevance of the levelized product cost for alternative market settings, including price taking firms, monopoly and oligopolistic competition.
1 Introduction

Numerous surveys and studies have suggested that managers frequently rely on a product’s full cost for a range of planning decisions, including product pricing. Economists have generally decried this practice as “irrational”, arguing that fixed costs are typically sunk and therefore irrelevant for decision making. In particular, many industrial organization models point to the dangers of double marginalization when managers internalize a cost higher than marginal cost (Sutton 1991, p. 28). The continued use of full cost for pricing and other short-run decisions has prompted some economists to advocate behavioral models, as alternatives to the paradigm of fully rational decision-making in order to explain managerial practices (Al-Najjar, Baliga, and Besanko (2008)).

The identification and measurement of marginal cost, however, is hardly without controversy (Pittman 2009; Carlton and Perloff 2005). Measuring marginal cost properly has been particularly problematic in antitrust investigations of excess profitability and monopoly pricing. Pittman (2009) provides a comprehensive survey of different strands of the industrial organization- and antitrust literature. He notes that by default many economic studies have used variable production as an effective proxy for marginal cost. Pittman succinctly summarizes the resulting tension as follows: “It is difficult to understand how a firm that sets prices at true marginal cost is able to survive as a going concern unless that true marginal cost includes the marginal cost of capital”

Accounting textbooks, such as Horngren, Datar, and Rajan (2012) or Zimmerman (2010), portray the importance of full cost as a benchmark for breaking even in terms of accounting profit. The corresponding measure of full cost typically includes charges related to depreciation and other sunk costs. For short-run decisions, like product pricing, these textbooks advocate the use of incremental costs which usually comprise variable cost and applicable op-

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1 According to the survey by Govindarajan and Anthony (1983) more than 80% of respondents use a product’s full cost when setting list prices. Comparative results were obtained in later studies by Shim and Sudit (1995), Drury, Osborne, and Tayles (1993), and Bouwens and Steens (2008). Similarly, the extensive survey evidence on intracompany pricing suggests that the prevalent form of transfer pricing in vertically integrated firms relies on measures of full costs. See, for instance, Eccles (1985). Cooper and Kaplan (1988) examine the use of full cost for product portfolio decisions.

2 Borenstein (2008) suggests that economies of scale offer a potential way out of the dilemma between marginal cost pricing and non-negative economic profits.

3 One of the prominent applications of activity-based costing, of course, is to allocate fixed costs among multiple products sharing the same production facility. See, for instance, Cooper and Kaplan (1988).
portunity costs. However, accounting textbooks generally do not identify contexts in which full cost can serve as the measure of relevant cost. For what is arguably among the most central long-run decisions, namely investing in a new production facility, accounting textbooks typically do not advocate an accrual-based measure as the relevant cost, but instead follow the standard corporate finance prescription to evaluate the stream of discounted cash flows associated with a particular decision. While we do not take issue with this prescription, our perspective in this paper is that it would be worthwhile to have a unit cost figure that is relevant for capacity investment decisions.

Some authors in the management accounting literature have sought to provide a rationale for full cost pricing in connection with capacity investment decisions. In particular, Banker and Hughes (1994) examine the relationship between support activity costs and optimal output prices in a classic one-period news-vendor setting. Capacity is not a committed resource in their setting, since it is chosen after the output price has been decided. Consequently, they find that the marginal cost of capacity is relevant for the subsequent pricing decision. Banker and Hughes model multiple support activities and show that an activity-based measure of unit cost provides economically sufficient information for pricing decisions. Banker, Hwang, and Mishra (2002) extend the Banker-Hughes setting to a multi-period model with upfront capacity commitment. Essential to their model is that optimal prices and capacity choices cannot be decoupled. They find that product costs are based on a measure of expected variable costs plus a charge for the utilization of fixed activity resources. The optimal capacity charge differs across periods and increases monotonically in expected demand for the resource in each period.

Göx (2002) identifies settings in which full-cost-based prices are optimal if either demand is certain or if demand is unknown and firms have to commit up front to the product prices they charge. In the context of his one-period model, Göx concludes that if capacity and pricing decisions must be made with the same information setting, capacity costs are effectively relevant costs. If demand is uncertain and prices can be adjusted concurrent demand conditions, Göx (2002) advocates the use of traditional marginal cost, which excludes capacity costs, as the relevant cost for pricing decisions. Finally, Balakrishnan and Sivaramakrishnan (1996) find that in a setting with hard capacity constraints (that is, firms cannot augment installed capacity by paying a surcharge), full cost does not provide sufficient information for product and capacity planning. Balakrishnan and Sivaramakrishnan (2002, p. 11) sum-
marize the tenor of these studies as follows: “In other words, full-cost measures potentially have economic value in simplifying planning and pricing problems.” We observe that in most of the studies summarized here, it is difficult to distinguish between long-term capacity decisions and subsequent short-term pricing and output decisions that firms seek to make optimally in the face of changing market conditions, subject to existing capacity constraints.

The focus of this paper is on a cost measure which we label Levelized Product Cost (LPC). Borrowing from the literature on electricity generation (which coined the term Levelized Cost of Electricity), we examine a cost measure that formally captures the following verbal definition: “the levelized cost of electricity is the constant dollar electricity price that would be required over the life of the plant to cover all operating expenses, payment of debt and accrued interest on initial project expenses, and the payment of an acceptable return to investors” (MIT 2007). Interestingly, there does not appear to be the generally accepted formula for calculating levelized costs in the electricity literature (Reichelstein and Yorston (2012)).

The preceding definition makes clear that LPC is defined entirely in terms of cash flows and calibrated as the minimum average price that investors would have to receive on average in order to justify investment in a particular production facility. We demonstrate him that LPC can be interpreted as a full cost much in the same way accounting textbooks typically calculate the unit full cost of a product. To align these measures, however, depreciation has to be calculated in a way that reflects the replacement cost value of assets (see, for instance, Rogerson (2008)). In addition, the measure of full cost has to include capital charges on the remaining book value of the capacity assets and the overall capacity charge (the sum of depreciation and computed capital charges) must be marked up by a tax factor. This mark-up accounts for the fact that the initial investment can be amortized only gradually over time for income tax purposes. The inclusion often imputed interest charges and tax factor that drives our measure of full cost beyond the usual calculation in cost accounting textbooks.

Given the break-even conceptualization of the LPC, one might expect that in a competitive market with price taking firms, the equilibrium market price will at least on average be equal to the levelized product cost. We identify conditions for this to be true, thus confirming

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4 Some authors have operationalized the LCOE concept as “Life-time energy” divided by “Life-time energy generated”. Reichelstein and Yorston (2012) argued that such a conceptualization is generally not compatible with the break-even nature of the levelized cost concept.
the interpretation of LPC as the long-run marginal cost. Our multi-period model allows for periodic shocks to market demand, with the consequence that firms will idle part of their productive capacity in unfavorable states of the world. The condition of zero economic profit in a competitive industry implies that the expected market price must be equal to the LPC regardless of the degree of volatility in aggregate market demand.

Our analysis demonstrates that when the volatility in aggregate market demand is “limited”, the aggregate industry capacity in equilibrium will correspond to the expected demand at the market price corresponding to the LPC. With “significant” volatility in market demand, we obtain the somewhat counterintuitive result that the aggregate capacity in equilibrium will be larger than that obtained in a setting with limited volatility. The intuitive argument for this finding is that while firms have to commit to the initial capacity investment, they have the option of idling parts of that capacity in subsequent periods in response to unfavorable market conditions. Since the market price is bounded below by the variable cost of production (the short-run marginal cost), the payoff structure for firms effectively looks like a call option and this option becomes more valuable with significant demand volatility.

For price setting firms, we also establish that the LPC is the relevant cost for making capacity investments. In particular, a monopolist facing limited demand uncertainty will choose capacity so that in each subsequent period marginal revenue is equal to LPC provided the available capacity is fully exhausted. This finding stands in partial contrast to the usual characterization in which capacity considerations are absent. With significant volatility in market demand, however, the relevant cost is effectively below the LPC, leading to a higher capacity investment and lower monopoly prices in expectation.

The characterization of LPC as long-run marginal cost is consistent with the findings in a number of recent accounting studies (Rogerson 2008; Rajan and Reichelstein 2009; Nezlobin 2012; Nezlobin, Rajan, and Reichelstein 2012). It is important to recall, however, that a key ingredient in those models is an infinite horizon of overlapping capacity investments. Capacity is then added continuously on an “as needed basis”. Furthermore, these models typically posit that market demand grows monotonically over time and is not subject to periodic shocks. As a consequence, firms never find themselves in a position of excess capac-

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5Our finding is broadly consistent with studies on activity-based costing which argue that cost allocation systems that capture the consumption of fixed resources provide a good estimate of a product’s long-run marginal cost Cooper and Kaplan 1988
ity. Our setting is motivated by the observation that in practice, the notion of a “lumpy” and irreversible capacity investment that may ex-post prove excessive under unfavorable demand conditions appears to be rather descriptive.\(^6\)

The remainder of the paper is organized as follows. Section 2 characterizes the Levelized Product Cost and its relation to the traditional cost accounting concept of full cost. Section 3 analyzes the equilibrium price and aggregate capacity level in a competitive market setting with price-taking firms. Price setting firms, in particular monopoly and oligopolistic competition, are the subject of Section 4. Finally, we conclude in Section 5.

## 2 Levelized Product Cost versus Full Cost

### 2.1 The LPC Concept

The levelized cost of a product or service seeks to account for all physical assets and resources required to deliver one unit of the product. Fundamentally, this concept seeks to identify a per unit break-even value that a producer would need to obtain as sales revenue in order to justify an investment in a particular production facility. As a consequence, the levelized product cost must be compatible with present value considerations for both equity and debt investors. In the context of electricity production, the concept introduced here is frequently referred to as the *Levelized Cost of Electricity*. In “The Future of Coal” (MIT 2007), the authors offer the following verbal definition:

*The levelized cost of electricity is the constant dollar electricity price that would be required over the life of the plant to cover all operating expenses, payment of debt and accrued interest on initial project expenses, and the payment of an acceptable return to investors*.

The Levelized Cost of Electricity concept is widely used by academic and business analysts to compare the cost effectiveness of alternative energy sources which differ substantially in terms of upfront investment cost and periodic operating costs. However, the formulaic implementation of this concept has been lacking in uniformity (Reichelstein and Yorston (2012)). In particular, some authors have conceptualized LCOE as the the ratio of “total

\(^6\)Balakrishnan and Sivaramakrishnan (2002) articulate this perspective as follows: *Organizations acquire some resources in lump sums up front and obtain other resources as needed. The former are “capacity” resources, and investments in theses resources constitute the fixed costs of running the business. Conversely, the costs of resources acquired when needed constitute the variable costs of production.*
lifetime cost” to total “lifetime electricity” produced. This turns out to be generally incompatible with the preceding verbal definition.

We formalize the levelized product cost concept in the context of a generic capacity investment in a new production facility. The levelized cost of one unit of output at this facility aggregates the upfront capacity investment, the sequence of output levels generated by the facility over its useful life, the periodic operating costs required to deliver the output in each period, and any tax related cash flows that apply to this type of facility. Since the output price in question should lead investors to break-even, specification of the appropriate discount rate is essential. We follow standard corporate finance theory in assuming that the project in question keeps the firm’s leverage ratio (debt over total assets) constant. The appropriate discount rate then is the Weighted Average Cost of Capital (WACC). In reference to the above quote in the MIT study, equity holders will receive “an acceptable return” and debt holders will receive “accrued interest on initial project expenses” provided the project achieves a zero Net Present Value (NPV) when evaluated at the WACC. We denote this WACC interest rate by \( r \) and the corresponding discount factor by \( \gamma \equiv \frac{1}{1+r} \).

Investment in the production facility may entail economies of scale. In particular,

- \( v(k) \): the cost of installing \( k \) units capacity. We normalize units so that one unit of capacity can produce one unit of output.
- \( T \): the useful life of the output generating facility (in years) and
- \( x_t \): the capacity decline factor: the percentage of initial capacity that is functional in year \( t \). Production in year \( t \) is limited to \( q_t \leq x_t \cdot k \).

The capacity decline factor reflects that in certain contexts the available output yield changes over time. For instance, with photovoltaic solar cells it has been observed that their efficiency diminishes over time. The corresponding decay is usually represented as a constant percentage factor \( x_t = x^{t-1} \) with \( x \leq 1 \) which varies with the particular technology (Reichelstein and Yorston, 2012). On the other hand, production processes requiring chemical balancing frequently exhibit yield improvements over time due to learning-by-doing effects, e.g., semiconductors and biochemical production processes. The analysis in this paper will pay particular attention to the “one-hoss shay” asset productivity scenario, in which the

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7See, for instance, Ross, Westerfield, and Jaffe (2005).
facility has undiminished capacity throughout its useful life, that is \( x_t = 1 \) for all \( 1 \leq t \leq T \) and thereafter the facility is obsolete\(^8\).

The unit cost of installed capacity, \( v(k) \), represents a joint cost of acquiring one unit of capacity for \( T \) years. In order to obtain the cost of capacity for one unit of output, the joint cost \( v(k) \) will be divided by the present value term \( \sum_{t=1}^{T} x_t \cdot \gamma^t \):

\[
c(k) = \frac{v(k)}{k \cdot \sum_{t=1}^{T} x_t \cdot \gamma^t}.
\]

We shall refer to \( c \) as the unit cost of capacity. Absent any other operating costs or taxes, \( c \) would yield the break-even price identified in the verbal definition above. To illustrate, suppose the firm makes an initial capacity investment of \( \$v(k) \) and therefore has the capacity to deliver \( q_t = x_t \cdot k \) units of product in year \( t \). If the revenue per unit is \( c \), then revenue in year \( t \) would be \( c \cdot x_t \cdot k \) and the firm would exactly break-even on its initial investment over the \( T \)-year horizon.\(^9\)

In addition to the initial investment expenditure \( v(k) \), the firm may incur periodic fixed operating costs. Our notation, \( F_t(k) \), indicates that the magnitude of these costs will generally vary with the scale of the initial capacity investment. Applicable examples here include insurance, maintenance expenditures, and property taxes. Unless otherwise indicated, we assume that the firm will incur the fixed operating cost \( F_t(k) \) regardless of the output level \( q_t \) it produces in period \( t \).\(^10\) The initial investment in capacity triggers a stream of future fixed costs and a corresponding stream of future (expected) output levels. By taking the ratio of these, we obtain the following time-averaged fixed operating costs per unit of output:

\[
f(k) \equiv \frac{T}{T} \frac{\sum_{t=1}^{T} F_t(k) \cdot \gamma^t}{k \cdot \sum_{t=1}^{T} x_t \cdot \gamma^t}
\]

With regard to variable production costs, we assume a constant returns to scale technology

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\(^8\)The one-hoss shay scenario is typical in the regulation literature, see for instance Laffont and Tirole (2000) and Rogerson (2011).

\(^9\)Throughout this section, it will be assumed that the available capacity is fully exhausted in each period, that is \( q_t = x_t \cdot k \). This specification will no longer apply in Sections 3 and 4, where uncertainty and demand shocks are introduced.

\(^10\)We will also consider an alternative scenario wherein the cost \( f_t(k) \) is incurred only if \( q_t > 0 \). Thus, the firm can avoid the fixed operating cost \( f_t(k) \) in period \( t \) if it idles the production facility in that period.
in the short run, so that the variable costs per unit of production up to the capacity limit are constant in each period, though they may vary over time. We again define a time-averaged unit variable cost by:

\[ w = \frac{\sum_{t=1}^{T} w_t \cdot x_t \cdot k \cdot \gamma^t}{k \cdot \sum_{t=1}^{T} x_t \cdot \gamma^t} \]  

(3)

Corporate income taxes affect the levelized cost measure through depreciation tax shields and debt tax shields, as both interest payments on debt and depreciation charges reduce the firm’s taxable income. While the debt related tax shield is already incorporated into the calculation of the WACC, the depreciation tax shield is determined jointly by the effective corporate income tax rate and the depreciation schedule that is allowed for the facility. These variables are represented as:

- \( \alpha \): the effective corporate income tax rate (in %),
- \( \hat{T} \): the facility’s useful life for tax purposes (in years), which is usually shorter than the projected economic life, i.e., \( \hat{T} < T \), and
- \( \hat{d}_t \): the allowable tax depreciation charge in year \( t \), as a % of the initial asset value \( v(k) \).

For the purposes of calculating the levelized product cost, the effect of income taxes can be summarized by a tax factor which amounts to a “mark-up” on the unit cost of capacity, \( c \).

\[ \Delta = \frac{1 - \alpha \cdot \sum_{t=1}^{\hat{T}} \hat{d}_t \cdot \gamma^t}{1 - \alpha} \]  

(4)

The tax factor \( \Delta \) exceeds 1 but is bounded above by \( \frac{1}{1-\alpha} \) if \( t = 0 \).\(^{11}\) It is readily verified that \( \Delta \) is increasing and convex in the tax rate \( \alpha \). Holding \( \alpha \) constant, a more accelerated tax depreciation schedule tends to lower \( \Delta \) closer to 1. In particular, \( \Delta \) would be equal to 1 if the tax code were to allow for full expensing of the investment immediately, that is, \( \hat{d}_0 = 1 \) and \( \hat{d}_t = 0 \) for \( t > 0 \).\(^{12}\)

\(^{11}\)To illustrate, for a corporate income tax rate of 35%, and a tax depreciation schedule corresponding to the double declining balance rule over 20 years, the tax factor would amount to roughly \( \Delta = 1.3 \).

\(^{12}\)Since the assumed useful life for tax purposes generally satisfies \( \hat{T} < T \), we will from hereon simply work refer to \( T \) as the useful life with the understanding that \( \hat{d}_t = 0 \) for \( \hat{T} \leq t \leq T \).
Combining the preceding components, we are now in a position to state the following characterization of the levelized product cost:

**Proposition 0** The Levelized Product Cost (LPC) is given by

\[
LPC(k) = w + f(k) + c(k) \cdot \Delta
\]

(5)

with \(c(k), w, f(k)\) and \(\Delta\) as given in (1) - (4).

To see that the expression in (5) does indeed satisfy the verbal break-even definition provided above, let \(p\) be the unit sales price. Figure 1 illustrates the sequence of annual pre-tax cash flows and annual operating incomes subject to taxation.

![Cash Flows and Taxable Income](image)

Taxable income in period \(t\) is given by the contribution margin in the respective period minus fixed operating costs minus the depreciation expense allowable for tax purposes:

\[
I_t = (p - w_t) \cdot x_t \cdot k - F_t(k) - d_t^o \cdot v(k).
\]

As the firm pays an \(\alpha\) share of its taxable income as corporate income tax, the annual after-tax cash flows, \(CF_t\) are:

\[
CF_0 = -v(k)
\]

and

\[
CF_t = (p - w_t) \cdot x_t \cdot k - F_t(k) - \alpha \cdot I_t.
\]

In order for the firm to break-even on this investment, the product price \(p\) must be such that the present value of all after-tax cash flows is zero. Solving the corresponding linear
equation yields $p = LPC$. In conclusion, the levelized product cost is a break-even price which includes three principal components: the (time averaged) fixed operating cost per unit of output produced, $f$, the (time averaged) unit variable cost, $w$, and the unit cost of capacity, $c$, marked-up by the tax factor $\Delta$. It should be noted that the LPC identified in Proposition 0 is entirely a cash flow concept. Depreciation for tax purposes enters only to the extent that there is an associated cash flow.

2.2 Relation to Full Cost

In the cost accounting literature, the concept of full cost is usually articulated as a unit cost measure that comprises variable production costs plus (allocated) overhead costs. Overhead costs, in turn, usually include both fixed and variable components. These components may contain accruals that arise due to cash expenditures being allocated cross-sectionally across products or inter-temporally across time periods, e.g., depreciation charges. This section explores to what extent a properly constructed measure of full cost can align with the levelized product cost identified above. In the context of a one-product firm, the traditional measure of full cost in period $t$ would be:

$$
\hat{FC}_t(k) = w_t \cdot q_t + F_t(k) + d_t \cdot v(k),
$$

(6)

where $q_t$ is the quantity produced in period $t$, $d_t$ is the percentage depreciation charge in period $t$, which may of course differ from the charge that is applicable for tax purposes. The total depreciation charge in period $t$ is given by $d_t \cdot AV_0 = d_t \cdot v \cdot k$, with $\sum d_t = 1$. It is intuitively obvious that a traditional full cost measure cannot capture the LPC since the latter is a discounted cash flow concept, yet (6) does not take the time value of money into consideration. This consideration leads to the following measure of fully loaded cost:

$$
FC_t(k) = \frac{w_t \cdot q_t + F_t(k) + [d_t + r \cdot (1 - \sum_{i=1}^{t-1} d_i)] \cdot v(k) \cdot \Delta}{q_t}.
$$

(7)

By construction, $FC_t > \hat{FC}_t$ in each period because the former concept includes an imputed capital charge on the remaining book value and a mark-up corresponding to the tax factor.

Proposition 1 Suppose the asset’s productivity profile conforms to the one-hoss shay scenario ($x_t = 1$), $w_t = w$, and $F_t(k) = f \cdot k$. With full capacity utilization, that is, $q_t = x_t \cdot k$,
fully loaded cost is equal to the levelized product cost in each period, that is,

\[ FC_t(k) = LPC(k), \]

provided depreciation is calculated according to the annuity method, that is, the depreciation schedule \( \{ d_t \} \) satisfies \( d_{t+1} = d_t \cdot (1 + r) \).

The claim in Proposition 1 relies on the well-known observation that with annuity depreciation:

\[ d_t + r \cdot (1 - \sum_{i=1}^{t-1} d_i) = \frac{1}{\sum_{i=1}^{T} \gamma^i}. \]

As a consequence, the last term in the numerator of (7) is equal to \( c(k) \cdot \Delta \), establishing the claim.

The conditions for Proposition 1 appear rather restrictive. A more general result can be obtained provided the measure full cost is based on additional accrual accounting concepts. First, suppose that either the unit variable costs \( w_t \) or the unit fixed operating costs \( F_t(k) \) change over time. In order for \( FC_t \) to align with LPC, the measure of full cost can no longer recognize variable and fixed operating costs at their current cash values, but instead on an accrual basis.\(^{13}\)

Secondly, the one hoss-shay assumption in Proposition 1 can be relaxed, provided depreciation is calculated according to the so-called Replacement Cost Rule. In particular, there exists a unique depreciation rule, \( (d_1, ..., d_T) \) such that for any productivity profile \( (x_1, ..., x_T) \):

\[ x_t \cdot c(k) = [d_t + r \cdot (1 - \sum_{i=1}^{t-1} d_i)] \cdot v(k). \]

The label replacement cost accounting indicates that for this depreciation rule the remaining book value at date \( t \),

\[ AV_t = v(k) \cdot [1 - \sum_{i=1}^{t} d_i] \]

would exactly be equal to the fair market value of a used asset if there was a competitive rental market for capacity services (Rogerson 2008).

\(^{13}\)These accounting rules also would have to be admissible for tax purposes.
Given the conditions of Proposition 1, we note that if the depreciation schedule \( \{d_t\} \) is accelerated relative to the benchmark of the annuity method, our expected measure of full cost in (7) will still be equal to the LPC on average in the sense that the present value of the two cost measures will be the same. In particular, the use of straight-line depreciation implies that the capital charges \( d_t + r \cdot (1 - \sum_{i=1}^{t-1} d_i) \) will increase over time. Thus, \( FC_t > LPC \) up to some date \( 1 \leq \hat{t} \leq T \), but \( FC_t < LPC \) thereafter. The common reliance on straight-line depreciation also motivates the following observation:

**Corollary 1** Suppose \( x_t = 1, w_t = w, \) and \( f_t(k) = f \cdot k \). With full capacity utilization, the levelized product cost exceeds full cost in each period:

\[
LPC(k) > FC_t(k)
\]

for \( 1 \leq t \leq T \), provided depreciation is calculated according to the straight-line rule.

The traditional accounting measure of full cost based on full straight line depreciation entails a capacity charge of \( \frac{v(k)}{T} \) per unit of capacity. In contrast, the LPC requires this charge to be \( \frac{v(k)}{\sum \gamma^t} \). The corollary simply notes that for \( \gamma < 1 \):

\[
T > \sum_{t=1}^{T} \gamma^t.
\]

The main point of this paper is that in a range of alternative market settings the levelized product cost, \( LPC \), should be seen as the relevant cost, or at least a benchmark for the relevant cost, in making capacity planning- and product pricing decisions. From an ex-ante perspective, that is, before the firm commits to any irreversible capacity investments, \( LPC \) captures the unit cost of production on average across the following \( T \) periods. In that sense, \( LPC \) can be interpreted as the long-run marginal cost and the long-run average cost, while the short-run marginal cost is given by the variable unit cost \( w_t \). Proposition 1 and the above corollary show that fully cost aligns with the long-run marginal cost, and this cost figure exceeds the traditional accounting measure of full cost.

Most of the economics literature has abstracted from irreversible capacity investments by assuming instead that firms can rent production capacity. Rental capacity then effectively becomes a consumable input, like labor and raw materials. With a rental market for “capital”, there is of course no distinction between long-run and short-run costs, as the firm can freely adjust all production inputs in any given period. Such a setting is nonetheless of
In order to compare the economist’s notion of marginal cost with the above LPC concept. In our notation, Carlton and Perloff (2005, p. 254) argue that marginal cost can be conceptualized as:

\[ MC = w + (r + \delta) \cdot v, \]

where \( \delta \) denotes “economic depreciation”. Carlton and Perloff (2005) posit that with a competitive market for capacity services, one unit of capacity rented for one period of time should trade for \((r + \delta) \cdot v\), if \( v \) is the constant unit price per unit of capacity (that is, \( v(k) = v \cdot k \)), \( r \) is the required rate of return and \( \delta \) reflects the physical decay rate of capacity. This specification is indeed compatible with the LPC formulation in Proposition 1 under the additional assumptions that assets are infinitely lived (\( T = \infty \)) and the decline in capacity follows a geometric pattern, that is, \( x_t = x^{t-1} \). The denominator in the expression for the unit cost of capacity \( c \) in (7) then amounts to:

\[ \sum_{t=1}^{\infty} x^{t-1} \cdot \gamma^t = 1 - x + r. \]

Therefore the unit cost of capacity \( c \) in (7) coincides with the marginal cost of capital in (8), provided the economic depreciation rate \( \delta \) is equated with \( 1 - x \), the capacity ’survival’ factor. This equivalence should not come as a surprise to the extent that both the LPC cost in Proposition 1 and the notion of a competitive rental market for capacity are pegged to an economic break-even condition. We note that Carlton and Perloff (2005) do not include fixed costs in their notion of marginal cost. In the context of our framework, the interpretation of LPC as the long-run marginal cost suggests the inclusion of the fixed costs \( F_t(k) \) since these costs will be incurred in future periods, yet they are “in play”, and therefore incremental, at date 0. For the same reasons, the tax factor \( \Delta \) should also be included in the calculation of long-run marginal cost.

In concluding this section, we observe that our framework identifies the relevant cost of production in a setting where an irreversible investment in capacity is required in order to produce a stream of future outputs. Once the investment decision has been made, the corresponding expenditure \((v(k))\) and subsequent operating fixed costs \((F_t(k))\) are sunk. In response to subsequent demand fluctuations for its product, the firm is then left to optimize within its capacity budget, with the unit variable cost, \( w_t \), left as the only relevant cost. In contrast, in Rogerson (2008), Rajan and Reichelstein (2009), Dutta and Reichelstein (2010)
and Nezlobin (2012), the firm makes a sequence of overlapping capacity investments. With an infinite planning horizon, a monotonically expanding market for its product and the absence of periodic shocks to demand, the firm never finds itself in a position with excess capacity. In effect, the sunk cost nature of past capacity investments never manifests itself in such a setting.

3 Price Taking Firms

This section examines the role of levelized product costs in a market setting with a “large” number of identical firms. In particular, firms are assumed to be price-takers, they have identical cost structures and there are no barriers to entry. For expositional simplicity, we suppose initially that the market for the product in question opens at date 0 (there are no market incumbents at date 0) and closes at date $T$, possibly because the current product or production technology will be replaced by a superior one at that point in time.

Figure 2 reproduces the standard textbook graph of equilibrium in a competitive industry. In equilibrium the market price must be equal to both marginal- and average cost. In particular, if a firm is to cover its periodic operating fixed costs so as to obtain zero-economic profits, marginal cost must be below the market price for some range of output levels below the equilibrium output level.¹⁴

Suppose now that, starting at date 0, suppliers can make irreversible capacity investments on the terms described in the previous section. Until the market closes at date $T$, firms can make additional capacity investments. In each period firms adjust prices and output to current demand conditions subject to their current capacity constraints. The expected aggregate market demand in period $t$ is given by $Q_t = D_o^p(p_t)$. The functions $D_o^p(\cdot)$ are assumed to be decreasing and we denote by $P_o^p(\cdot)$ the inverse of $D_o^p(\cdot)$. The actual price in period $t$ is a function of the aggregate supply $Q_t$ and the realization of a random shock $\tilde{\epsilon}_t$:

$$P_t(Q_t, \tilde{\epsilon}_t) = \tilde{\epsilon}_t \cdot P_o^p(Q_t).$$ (9)

¹⁴Borenstein (2000) articulates this point as follows: “It is important to understand that a price-taking firm does not sell its output at a price equal to the marginal cost of each unit of output it produces. It sells all of its output at the market price, which is set by the interaction of demand and all supply in the market. The price-taking firm is willing to sell at the market price any output that it can produce at a marginal cost less than that market price.”
The specification of multiplicatively separable shocks will be convenient in order to quantify a threshold value for the magnitude of the periodic uncertainty.\textsuperscript{15} The random variables $\tilde{\epsilon}_t$ are assumed to be serially uncorrelated and to have densities $h_t(\cdot)$ on a common support $[\xi, \bar{\epsilon}]$ with $1 > \xi > 0$ and $\bar{\epsilon} > 1$. In order for $P^o_t(\cdot)$ to be interpreted as the expected inverse demand curve, we also normalize the periodic random fluctuations such that:

$$E[\tilde{\epsilon}_t] = \int_\xi^{\bar{\epsilon}} \epsilon_t \cdot h_t(\epsilon_t) \, d\epsilon_t = 1.$$  

Firms in the industry are assumed to be risk neutral and to have the same information regarding future demand.\textsuperscript{16} In particular, they anticipate that $\tilde{\epsilon}_t$ will be realized at the beginning of period $t$, prior to each supplier deciding its current level of output, less than or equal to its capacity level. Provided each firm is a price taker, the supply curve in period $t$

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\textsuperscript{15}In contrast, the research reviewed by Balakrishnan and Sivaramakrishnan (2002) on capacity choice and full cost pricing exclusively considers an additive error term for the aggregate demand function.

\textsuperscript{16}We do not consider inventory build-ups in our model. In some industries, holding inventory is not a viable option because of high inventory holding costs (e.g., electricity). The general effect of low inventory holding costs will be to smooth out demand fluctuations. As a consequence, we would then expect the equilibrium prices and capacity levels to be close to those obtained under conditions of demand certainty. Finally, we note that the earlier literature on capacity planning and full cost pricing has also ignored the possibility of inventory build-ups (Balakrishnan and Sivaramakrishnan (2002)).
is illustrated by the sequence of vertical and horizontal red lines in Figure 3. In particular, a firm exhausts its full capacity whenever the market price covers at least the short-run marginal cost $w_t$.

![Figure 3: Competitive Supply Curve with Capacity Constraint](image)

To develop the results in this section, it will be convenient to begin with a long-run constant returns to scale technology. Thus $v(k) = v \cdot k$ (and therefore $c(k) = c \cdot k$) and $F_t(k) = f_t \cdot k$. As a consequence the levelized product cost $LPC(k)$ is therefore independent of $k$ and will be denoted simply by $LPC^{17}$. Initially, we shall also focus on a time-invariant cost- and capacity structure such that $x_t = 1$ $w_t = w$, $h_t(\cdot) = h(\cdot)$, and $f_t(k) = f(k)$ for $1 \leq t \leq T$. Furthermore, we consider first a setting where the expected aggregate demand is unchanged over the $T$-period horizon, that is $P_t^0(\cdot) = P^0(\cdot)$. We refer to the combination of these assumptions as a stationary environment.

Since the levelized product cost is the threshold price at which firms break-even on their capacity investments, one would expect that, with frictionless entry into the industry, the competitive equilibrium price will at least “on average” be equal to the long-run marginal cost. The following result confirms this intuition and characterizes the aggregate level of capacity in equilibrium. We denote by $K_t$ the aggregate capacity level in the industry and

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17The assumption of fixed operating costs that are unavoidable (even if $q_t = 0$) is of obvious importance here.
by $P_t(w, \epsilon_t, K)$ the equilibrium price in period $t$, contingent on the current unit variable cost, the realization of $\epsilon_t$, and the aggregate industry capacity level $K$.

**Proposition 2** Given a stationary environment, an equilibrium implies a unique aggregate capacity level $K^*$, such that the expected product price satisfies:

$$E[P(w, \tilde{\epsilon}_t, K^*)] = LPC.$$  

The equilibrium capacity level $K^*$ is bounded by:

$$D^o\left(\frac{LPC}{\epsilon}\right) \geq K^* \geq D^o(LPC),$$

(10)

with $K^* = D^o(LPC)$ if and only if $\text{Prob}\left[\tilde{\epsilon} \geq \frac{w}{LPC}\right] = 1$.

Proposition 2 reinforces the interpretation of LPC as the long-run marginal cost. Absent any shocks to demand, that is, $P(Q_t, \tilde{\epsilon}_t) = P^o(Q_t)$, firms will in each period produce at full capacity provided $P^o(K^*) \geq w_t$. The capacity constraint the industry from bidding the market price down to $w$. At the investment stage, the condition of zero economic profits for each firm dictates that the aggregate capacity level must satisfy $P^o(K^*) = LPC$. The stationarity of the environment implies that in equilibrium all capacity investments will be made initially at date 0. In other words, in equilibrium all firms move in lock-step with their investments at date zero and effectively foreclose the possibility of entry in the remaining $T - 1$ periods. Figure 4 illustrates the finding in Proposition 2.

The equilibrium capacity level corresponds to the expected equilibrium price, that is $P^o(K^*) = LPC$, if market demand is subject to limited volatility. Hence, the result in Proposition 2 extends to uncertain environments provided the magnitude of the periodic shocks to aggregate demand is not “too large”. The formal meaning of not “too large” in (15) is that $\epsilon \cdot LPC \geq w$, or equivalently:

$$\text{Prob}[\tilde{\epsilon} \geq \frac{w}{LPC}] = 1.$$  

(11)

This condition holds if an unfavorable shock can result in a percentage deviation from average demand that is bounded in magnitude by the ratio between the unit variable cost (short-run marginal cost) and the levelized product cost. We note that this condition is more likely to be satisfied in capital intensive industries characterized by high capacity investment.
Provided condition (15) is met, firms in the industry will exhaust the available capacity even for unfavorable realizations of the noise term $\tilde{\epsilon}$.

Intuitively, one might expect that the effect of demand variability is to result in a lower aggregate capacity levels than $K^o \equiv D^o(LPC)$, because if condition (15) is not met, some of the industry’s aggregate capacity will be idle with positive probability. Individual firms are indifferent as to whether they idle part of their capacity since the equilibrium market price will drop to $w$ once the aggregate capacity is not fully utilized\(^{19}\). However, the condition of zero economic profits dictates that the aggregate capacity will be larger than under conditions of certainty because firms do not need to commit the entire LPC at the investment stage. In particular, firms have a *call option* to idle parts of their capacity which allows them to

\(^{18}\)Applicable examples include the semiconductor industry, electricity production and air travel.

\(^{19}\)An implicit assumption here is that $\epsilon \cdot P^o(0) \geq w_t$. 

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Figure 4: *Aggregate Equilibrium Capacity in a Competitive Equilibrium*
avoid the variable cost in case of unfavorable demand realizations. This call option becomes more valuable with higher volatility, that is, the $\bar{\epsilon}$ exhibits a larger variance.

The notion of acquiring costly capacity as a call option can be illustrated further by the following comparative statics exercise. Suppose the unit variable cost $w$ increases and either the cost of capacity $c$ or the fixed cost $f$ decreases by a corresponding amount such that the overall LPC stays constant. Thus the cost of acquiring the option decreases (to the extent $c$ decreases), yet at the same there is a higher chance of the firm receiving the lower bound payoff of zero. In particular, we note that if the unit variable cost increases by $\Delta w$, the aggregate equilibrium capacity level, $K^*$ will increase provided condition (15) is not met at $w$. In contrast, $K^*$ would stay unchanged, and equal to $D^o(LPC)$, if (15) is met at $w + \Delta w$.

The preceding argument is useful in order to address the impact of avoidable fixed operating cost in case the firm idles its entire capacity and does not produce any output in period $t$. The result in Proposition 2 then remains valid as stated, except that the aggregate level of capacity may increase. If the fixed cost $f$ is avoidable, the price floor in each period becomes $w + f$ instead of $w$ because suppliers would withhold capacity if the price were to drop below $w + f$.

**Corollary 2** If the periodic operating fixed costs per unit of capacity, $f$, are avoidable, the expected equilibrium price remains equal to LPC. The equilibrium level of aggregate capacity increases relative to the scenario where $f$ is unavoidable if and only if

$$\text{Prob}[\bar{\epsilon} \geq \frac{w + f}{LPC}] < 1.$$ 

Proposition 2 allows us to tie competition to reported profitability, in particular the expected net income that firms in a competitive industry will report in equilibrium. In the context of our model, net-income will be defined as operating income less taxes paid. By the conservation property of residual income (Preinreich (1938)), the present value of future residual incomes will be zero. However, the inter-temporal distribution of the net income and residual income numbers obviously depends on both the financial- and tax reporting rules for operating assets. In light of the prevalence of straight-line rule depreciation for financial reporting purposes, the following result assumes that this rule applies.

**Corollary 3** Suppose firms use straight-line depreciation for financial reporting purposes. In equilibrium, expected net-income will be positive in each period, regardless of the depreciation rule used for tax purposes, provided
\[
\alpha \leq 1 - \frac{\sum_{i=1}^{T} \gamma^i}{T}.
\] (12)

The sufficient condition in (12) is based on the lower bound scenario in which the initial investment is directly expensed for tax purposes. Even then, equation (12) will be satisfied for many reasonable parameter constellations. Ceteris paribus, it will be met for a higher discount rate and investment projects with a longer useful life. A sharper prediction regarding net-income can be obtained if one assumes that straight-line depreciation is applied for both financial reporting and tax purposes. We then obtain the following prediction for expected net income per unit of capacity acquired:

\[
\pi = (1 - \alpha)[LPC - w - f - \frac{v}{T}] = (1 - \alpha) \left[ v \left( \frac{\Delta}{\sum_{i=1}^{T} \gamma^i} - \frac{1}{T} \right) \right] > 0.
\] (13)

Dividing the right-hand side of (13) by \( v \) then yield the expected after-tax profit per dollar of initial investment for a firm in a competitive industry.

The remainder of this section seeks to relax some of the assumptions underlying Proposition 2. One consequence of the assumed constant returns to scale technology in our model has been that the efficient scale of operation for individual firms, that is the individual \( k \), remains indeterminate. If the cost of acquiring capacity \( v(k) \) and the periodic fixed operating costs \( F_t(k) \) are non-linear, the condition of zero expected economic profits dictates that the efficient scale of operation must be chosen such that the LPC per unit of output is minimized. Referring back to the generalized definition of LPC(s) is Section 2, we define the efficient scale of operation by:

\[
k^* \in \arg\min\{LPC(k)\}.
\]

The expected equilibrium price in Proposition 2 then becomes \( LPC(k^*) \) while the characterization of the aggregate capacity level \( K^* \) is unchanged.

We now examine a setting in which the aggregate demand is no longer stationary but instead contracts over time.\(^{20}\) In particular, suppose that \( P_t^c(\cdot) = \lambda_t \cdot P^o(\cdot) \) for all \( 1 \leq t \leq T \)

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\(^{20}\)Recent work on capacity investments, such as Rogerson (2008, 2011), Rajan and Reichelstein (2009), Nezlobin (2012) and Nezlobin, Rajan, and Reichelstein (2012), has consistently focused on expanding markets, so that firms are never left with excess capacity.
such that $\lambda_{t+1} < \lambda_t < 1$ for all $1 \leq t \leq T$. We refer to such a scenario as a *declining* product market. Define:

$$m(\lambda) = \frac{\sum_{t=1}^{T} \gamma^t \lambda_t}{\sum_{t=1}^{T} \lambda_t \cdot \gamma^t}$$

Clearly, $m(\lambda) > 1$ in a declining product market. Intuitively, one would expect the equilibrium capacity in a declining product to be lower compared to a stationary market. The following result confirms this intuition and shows that 'on average' the expected equilibrium market price will also be higher as a consequence of market demand that diminishes over time.

Let $P_t(w, \epsilon_t, \lambda_t, K)$ denote the equilibrium market price in period $t$ when aggregate market demand is given by $\lambda_t \cdot P^o(\cdot)$ and the current demand shock is realized. By construction,

$$P_t(w, \epsilon_t, \lambda_t, K) = \begin{cases} w & \text{if } \epsilon_t < \epsilon(w, K, \lambda_t) \\ \lambda_t \cdot P^o(K) & \text{if } \epsilon_t > \epsilon(w, K, \lambda_t), \end{cases}$$

with

$$\epsilon(w, K, \lambda_t) \cdot \lambda_t \cdot P^o(K) \equiv w.$$ 

We introduce the notation $\lambda = (\lambda_1, ..., \lambda_T)$.

**Proposition 3** With a declining product market, there exists a unique aggregate capacity level $K^*$ such that the expected equilibrium product prices satisfy:

$$\sum_{t=1}^{T} E[P_t(w, \epsilon_t, \lambda_t, K)] \cdot \gamma^t = LPC \cdot m(\lambda) \sum_{t=1}^{T} \gamma^t.$$

The equilibrium capacity level $K^*$ is bounded by:

$$D^o\left(\frac{1}{\hat{\epsilon}} \cdot LPC \cdot m(\lambda)\right) \geq K^* \geq D^o(LPC \cdot m(\lambda)) \quad (14)$$

with $K^* = D^o(LPC \cdot m(\lambda))$ if and only if:

$$\text{Prob} \left[ \hat{\epsilon} \geq \frac{w}{LPC \cdot m(\lambda)} \right] = 1. \quad (15)$$

In case of a growing product market, $\lambda_{t+1} > \lambda > 1$, Proposition 2 also remains valid provided the growth rate is 'sufficiently small'. Proposition 3 can also extended to include variable costs that change over time and time-variant distributions for $\hat{\epsilon}_t$. 

22
4 Price Setting Firms

4.1 Monopoly

We now turn to a monopolistic firm that faces the same stationary environment posited in connection with Proposition 2. In particular, suppose that expected aggregate market demand in period \( t \) is given by \( q_t = D^o(p_t) \). The function \( D^o(\cdot) \) is assumed to be decreasing and we denote by \( P^o(\cdot) \) again the inverse of \( D^o(\cdot) \). The actual price in period \( t \) is a function of the quantity supplied \( q_t \) and the realization of the random shock \( \tilde{\epsilon}_t \):

\[
P(q_t, \tilde{\epsilon}_t) = \tilde{\epsilon}_t \cdot P^o(q_t).
\]

(16)

Similar to our assumptions in Section 3, the monopolist observes the realization of \( \tilde{\epsilon}_t \) before deciding on the quantity supplied in that period. Let \( MR^o(q) \) denote the marginal revenue, with

\[
MR^o(q) \equiv \frac{d}{dq} [P^o(q) \cdot q].
\]

Assuming the short-run variable costs are again constant over time over time and equal to \( w \) and fixed operating costs in each period are avoidable and equal to \( f \) per unit of capacity, we obtain the following result:

**Proposition 4** The optimal capacity investment, \( k^* \), for a monopolist satisfies \( k^* \in [k^o, k^+] \), where

\[
MR^o(k^o) \equiv LPC \text{ and } MR^o(k^+) \equiv \frac{LPC}{\tilde{\epsilon}}.
\]

(17)

Furthermore, \( k^* = k^o \) if and only if

\[
Prob \left[ \tilde{\epsilon} \geq \frac{w}{LPC} \right] = 1.
\]

(18)

Proposition 5 reinforces our general claim that the LPC is the relevant cost for capacity planning purposes, at least if market demand is subject to limited volatility (condition (18 holds). Given our finding in Proposition 2, we also conclude that with limited volatility in market demand \( k^* < K^* \), i.e., the monopolist’s capacity investment is lower than the aggregate capacity level that obtains under competition. With limited demand volatility, we also confirm the usual prediction that the monopoly price entails a mark-up over the
“relevant” cost and the mark-up is determined by the price elasticity of demand according to the rule:

\[ p^*(k^*, \epsilon_t) = \epsilon_t \cdot LPC \cdot \frac{E(k^*)}{E(k^*) - 1}, \]

where \( E(k^*) > 1 \) denotes the price elasticity of demand. In contrast to the customary textbook solution, though, the basis for the mark-up is now the levelized product cost rather than the shot-run variable cost \( w \). Once demand volatility becomes significant, in the sense that condition (18 does not hold, the monopolist will withhold capacity for unfavorable realizations of market demand. In those states, the relevant cost then, of course, becomes again \( w \).

Holding all other variables fixed, Proposition 5 indicates that the monopoly capacity investment will be higher as volatility moves from “limited” to “significant”. This would, in turn, suggest lower expected monopoly prices. To formalize this comparative statics result, we refer to the standard definition which posits that \( \epsilon^1 \) exhibits higher volatility than \( \epsilon^2 \) if \( h^1(\cdot) \) can be expressed as a mean-preserving spread of \( h^2(\cdot) \).

**Corollary 4 (Conjecture)** Higher volatility in market demand results in a higher capacity investment and lower expected monopoly price.

To conclude this section, we note that, in contrast to our finding in Corollary 2 in Section 2, the avoidability of fixed operating costs will generally have no impact on the monopolist’s capacity investment. In a competitive setting, the importance of avoidability of the fixed cost \( f \) was that the price would not drop below \( w + f \). For monopoly pricing, however, the presence of the fixed operating cost will not affect the price charged unless unfavorable demand realizations would drive the firm to shut down production completely in a given period.

### 4.2 Oligopoly

Our findings in the previous two sections have demonstrated that the levelized product cost (LPC) should be viewed as the relevant cost, or at least as a benchmark for the relevant cost, both for a monopoly and a competitive setting. This suggests that this characterization of LPC also carries over to oligopolistic settings in which the incumbent firms make initial
capacity investments and subsequently compete subject to their own capacity constraints. We consider both quantity competition (Cournot) and price competition in the sense of Bertrand.

We begin with quantity competition in which firms first make simultaneous capacity decisions and then in each period choose production quantities simultaneously which in turn determines the current market price. In the simplest setting, there are \( m \) identical firms facing a stationary environment as described in Section 3. If the firms choose the quantities \( q_t = (q_1^t, ... q_m^t) \) the product price in period \( t \) is given by:

\[
P(Q_t, \tilde{\epsilon}_t) = \tilde{\epsilon}_t \cdot P^\alpha(\sum_{i=1}^{m} q_i^t).
\]

(19)

The initial capacity investments are denoted by \( k = (k_1, ..., k_m) \). Thus aggregate capacity is given by

\[
K(k) = \sum_{i=1}^{m} k_i
\]

(20)

We also use the common notation \( Q^{-j} = \sum_{i \neq j} q_i \) and \( K^{-j}(k) = \sum_{i \neq j} k_i \). The marginal revenue from the perspective of the i-th firm given the quantity and capacity choices of the other participants is given by:

\[
MR^\alpha(q^i|Q^{-i}) \equiv \frac{d}{dq^i}[P^\alpha(Q) \cdot q^i].
\]

**Proposition 5** Suppose that condition

\[
Prob \left[ \tilde{\epsilon} \geq \frac{w}{LPC} \right] = 1.
\]

(21)

holds. Then the following constitutes a subgame perfect equilibrium:

i) At date 0, firms choose identical capacity investments, \( k^* \), which satisfy the equation:

\[
MR^\alpha(k^*|(n-1) k^*) \equiv LPC
\]

(22)

\[
ii) In subsequent periods \( 1 \leq t \leq T \), each firm supplies the maximal production quantity \( q_i^t = k^* \).
\]

Clearly, Proposition 5 only addresses the scenario of limited demand volatility in each period. Consistent with the other competition settings examined above, we then conclude that the levelized product cost, LPC, is unambiguously the relevant unit cost for capacity
planning decisions. Propositions 2 and 4 also suggest that with significant volatility that relevant cost will be below the LPC.

Finally, it would be of interest to understand the outcome under price, rather than quantity competition at the stage games played at dates 1 through T. Given the presence of capacity constraints, the results of Kreps and Scheinkman (1983) suggest that the unique equilibrium outcome may, in fact, not be different from the Cournot outcome characterized in Proposition 5.

5 Concluding Remarks

Investment in productive capacity are one of the most far reaching and strategic decisions for many firms. The scope of our analysis is to identify the relevant cost for capacity decisions. Economists typically point to marginal cost as the relevant cost for capacity and pricing decisions. Accounting textbooks recognize full cost as an important benchmark if firms are to survive in the long run, however, they do not offer a rationale for full costing in decision-making. For capacity investment decisions, the usual prescription is to evaluate projects based on discounted cash flows. Some studies in the management accounting literature have sought to identify environments in which full cost can indeed serve as the basis for decision making. However, these models typically do not distinguish between the long-run perspective for capacity investments and a subsequent short-run perspective when firms choose prices and production quantities subject to the available capacity.

Our model establishes the Levelized Product Cost (LPC) as the relevant cost for capacity decisions. By construction, the LPC is a break-even price which comprises the unit cost of capacity marked up by a tax factor and includes time-averaged fixed operating costs per unit of output in addition to time-averaged unit variable costs. The LPC can be obtained as a measure of fully loaded provided depreciation is calculated in a manner that reflects the decline in productive capacity. In addition, full cost must include capital charges on the outstanding book value of the productive asset and the pre-tax charge for capacity must be marked up by a tax factor which reflects the depreciation schedule allowable tax purposes.

We show that the LPC corresponds to the expected market price in a competitive market setting, thus confirming the interpretation of LPC as long-run marginal cost. Moreover, with significant volatility in market demand, the aggregate capacity level in a competitive industry will exceed the level given by the expected demand at LPC, as firms do not need to commit
to use their entire capacity but retain a call option to idle parts of their installed capacity in future periods. In a monopoly setting, LPC is still the relevant cost with limited volatility in market demand. We find that, ceteris paribus, higher volatility tends to result in more capacity and lower expected monopoly prices. As one would expect, equilibrium capacity in a monopoly will be lower than in a competitive industry. Overall, the measure of levelized product cost integrates the accountant’s concept of full average cost with a unit cost figure that is relevant for acquiring long-term and irreversible capacity resources.
6 Appendix

Proof of Proposition 2.

Denote the aggregate capacity investment at date 0 by $K$. We first characterize the equilibrium level of $K^*$ and the expected equilibrium prices, $E[P(w, \tilde{\epsilon}_t, K^*)]$, under the assumption that no firm makes additional capacity investments at any date $t$, $1 \leq t < T$. We subsequently confirm that in equilibrium there are indeed no capacity investments after date 0.

If the aggregate capacity level is $K$ at date 0, the equilibrium price in period $t$ is given by:

$$P(w, \epsilon_t, K) = \begin{cases} \epsilon_t \cdot P^0(K) & \text{if } \epsilon_t \geq \epsilon(K, w) \\ w & \text{if } \epsilon_t < \epsilon(K, w) \end{cases}$$

Here $\epsilon(K, w)$ denotes the unique cut-off level defined as follows:

$$\epsilon(K, w) = \begin{cases} \bar{\epsilon} & \text{if } \bar{\epsilon} \cdot P^0(K) \leq w \\ \frac{w}{P^0(K)} & \text{if } \bar{\epsilon} \cdot P^0(K) > w > \epsilon \cdot P^0(K) \\ \epsilon & \text{if } \epsilon \cdot P^0(K) \geq w \end{cases}$$

Clearly, $\epsilon(K, w)$ is weakly increasing in $K$.

Consider now an individual firm that has invested a capacity level $k$. Given the one-hoss shay productivity pattern, there is no loss of generality in assuming that this firm will indeed produce $k$ units of output since the product market price does not fall below the short-run managerial cost $w$. This follows from our assumption that

$$\epsilon \cdot P^0(0) \geq w_t.$$ 

Obviously, individual firms are indifferent about idling part as all of their capacity in case $P(w, \epsilon_t, K) = w$. The following arguments would lead to the same conclusion if one were to assume that the firm chooses some $0 \leq q \leq k$ in case the market price equals $w$ in period $t$.

The present value of expected future cash flows for the representative firm then becomes:

$$\Gamma(k|K) = \sum_{t=1}^{T} \left\{ E \left[ [P(w, \tilde{\epsilon}_t, K^*) - w_t] \cdot q - f \cdot k - \alpha \cdot \tilde{I}_t \right] \right\} \gamma^t - v \cdot k.$$ 

Here $\tilde{I}_t$ denotes taxable income given by:

$$\tilde{I}_t = [P(w, \tilde{\epsilon}_t, K) - w_t] \cdot q - f \cdot k - \tilde{d}_t \cdot v \cdot k.$$
In a competitive equilibrium the aggregated capacity level must be chosen so that for each atomistic firm: \( \Gamma(k|K^*) = 0 \). Solving this equation for the expected market price under the assumption that \( q = k \), the expected profit per unit of capacity acquired must be zero. Thus:

\[
(1 - \alpha) \sum_{t=1}^{T} E[P(w, \tilde{\epsilon}_t, K^*)] \cdot \gamma^t = \\
v \left[ 1 - \alpha \sum_{t=1}^{\hat{T}} \hat{d}_t \cdot \gamma^t \right] - \sum_{t=1}^{T} \{f + w\} \cdot \gamma^t.
\]

Dividing by \( (1 - \alpha) \) in (23) and recalling the definition of the tax factor, \( \Delta \):

\[
\Delta = 1 - \sum_{t=1}^{\hat{T}} \hat{d}_t \cdot \gamma^t
\]

as well as the definitions of the time-averaged unit fixed cost, \( f \) and the time-averaged unit variable cost \( w \), respectively, we conclude that the right-hand side of (23) is exactly equal to \( LPC \cdot \sum_{t=1}^{T} \gamma^t \) and therefore:

\[
E[P(w, \tilde{\epsilon}_t, K^*)] = LPC
\]

The expected equilibrium price in period \( t \) is equal to:

\[
E[P(w, \tilde{\epsilon}, K^*))] = \int_{\epsilon(K^*, w)}^{\epsilon(K^*, w)} w \cdot h(\epsilon) \, d\epsilon + \int_{\epsilon(K^*, w)}^{\epsilon} \epsilon \cdot P^0(K^*) h(\epsilon) \, d\epsilon.
\]

We next demonstrate that \( E[P(w, \tilde{\epsilon}, \cdot)] \) is decreasing in \( K \). For any \( K_1 < K_2 \), we can write \( E[P(w, \tilde{\epsilon}, K_1)] \) as:

\[
\int_{\epsilon(K_1, w)}^{\epsilon(K_1, w)} w \cdot h(\epsilon) \, d\epsilon + \int_{\epsilon(K_1, w)}^{\epsilon(K_2, w)} \epsilon \cdot P^0(K_1) \, h(\epsilon) \, d\epsilon + \int_{\epsilon(K_1, w)}^{\epsilon} \epsilon \cdot P^0(K_1) \, h(\epsilon) \, d\epsilon
\]

and similarly \( E[P(w, \tilde{\epsilon}, K_2)] \) can be expressed as:

\[
\int_{\epsilon}^{\epsilon(K_2, w)} w \cdot h(\epsilon) \, d\epsilon + \int_{\epsilon(K_1, w)}^{\epsilon(K_2, w)} \epsilon \cdot P^0(K_2) \, h(\epsilon) \, d\epsilon + \int_{\epsilon(K_1, w)}^{\epsilon(K_2, w)} \epsilon \cdot P^0(K_1) \, h(\epsilon) \, d\epsilon
\]

Since \( \epsilon \cdot P^0(K_1) \geq w \) for \( \epsilon(K_1, w) \) and \( P^0(K_1) > P^0(K_2) \), we conclude that each of the three integrals in (31) is respectively equal to or larger than its counterpart in (32).

Suppose next that the condition

\[
\text{Prob} \left[ \tilde{\epsilon}_t \geq \frac{w}{LPC} \right] = 1
\]
is met. By construction, we then have \( \epsilon(K^*, w) = \bar{\epsilon} \) and therefore
\[
E[P(w, \bar{\epsilon}, K^*)] = P^o(K^*).
\]

Equation (24) yields that \( K^* = K^0 \), with \( K^0 \) defined as \( P^o(K^0) \equiv LPC \). Conversely, if (28) is not met, then the equilibrium capacity level must satisfy \( K^* > K^0 \). To see this, assume to the contrary that \( K^0 < K^* \). If then follows from (30) that
\[
E[P^o(w, \bar{\epsilon}t, K^*)] \geq E[P(w, \epsilon t, K^0)] > P^o(K^0) \equiv LPC.
\]

The strict inequality in (29) follows from the fact that if the condition in (19) is not met, then \( \epsilon < \epsilon(K^0, w) < \bar{\epsilon} \). We thus conclude that \( K^* > K^0 \).

The show the upper bound on \( K^* \) claimed in Proposition 2, we note that the zero-profit condition again stipulates that:
\[
E[P(w, \bar{\epsilon}, K^*)] = w + f + c \cdot \Delta.
\]
It follows from (18) that
\[
E[P(w, \bar{\epsilon}, K^*)] \leq \bar{\epsilon} \cdot (1 - H(\epsilon(K^*, w))) \cdot P^o(K^*) + w \cdot H(\epsilon(K^*, w))
\]
where \( H(\cdot) \) is the cumulative distribution function of the density \( h(\cdot) \). Thus
\[
[\bar{\epsilon} \cdot P^o(K^*) - w] \cdot [1 - H(\epsilon(K^*, w))] \geq f + c \cdot \Delta
\]
and therefore
\[
P^o(K^*) \geq \frac{LPC}{\bar{\epsilon}},
\]
which is equivalent to \( K^* \leq D^o \left( \frac{LPC}{\bar{\epsilon}} \right) \).

It remains to show that in equilibrium all investments will occur immediately, that is no firm will make capacity investments at a date \( t \geq 1 \).
Proof of Corollary 2. If fixed operating costs are avoidable and the aggregate capacity level is equal to $K$, the expected equilibrium price in period $t$ is equal to:

$$E[P(w', \bar{\epsilon}, K)] = \int_{\xi}^{\epsilon(K, w')} w' \cdot h(\epsilon) \, d\epsilon + \int_{\epsilon(K, w')}^{\bar{\epsilon}} \epsilon \cdot P^o(K) h(\epsilon) \, d\epsilon. \quad (30)$$

where $w' \equiv w + f$. We note that $\epsilon(K, w')$ is weakly increasing in $w'$. Similar to the argument in the proof of Proposition 2, we next demonstrate that $E[P(w', \bar{\epsilon}, \cdot)]$ is increasing in $w'$. For any $w'_1 < w'_2$, we can write $E[P(w'_1, \bar{\epsilon}, K)]$ as:

$$\int_{\xi}^{\epsilon(K, w'_1)} w'_1 \cdot h(\epsilon) \, d\epsilon + \int_{\epsilon(K, w'_1)}^{\epsilon(K, w'_2)} \epsilon \cdot w'_1 \, h(\epsilon) \, d\epsilon + \int_{\epsilon(K, w'_2)}^{\bar{\epsilon}} \epsilon \cdot P^o(K) \, h(\epsilon) \, d\epsilon. \quad (31)$$

Similarly $E[P(w, \bar{\epsilon}, K_2)]$ can be expressed as:

$$\int_{\xi}^{\epsilon(K, w_2')} w_2' \cdot h(\epsilon) \, d\epsilon + \int_{\epsilon(K, w_2')}^{\epsilon(K, w_2)} \epsilon \cdot P^o(K) \, h(\epsilon) \, d\epsilon + \int_{\epsilon(K, w_2)}^{\bar{\epsilon}} \epsilon \cdot P^o(K) \, h(\epsilon) \, d\epsilon \quad (32)$$

Since $\epsilon \cdot P^o(K) \geq w$ for $\epsilon > \epsilon(K, w)$ and $w'_1 < w'_2$, we conclude that each of the three integrals in (31) is respectively at least as large as its counterpart in (32). It should be noted that the claimed monotonicity here is strict whenever $\epsilon(K, w'_2) > \xi$.

Denote the equilibrium capacity levels with avoidable and unavoidable fixed costs by $K^*$ and $K^{**}$, respectively, it follows directly from the arguments in Proposition 2 that:

$$E[P(w, \bar{\epsilon}, K^*)] = E[P(w + f, \bar{\epsilon}, K^{**})] = LPC. \quad (33)$$

Since $E[P(w, \bar{\epsilon}, K)]$ is decreasing in $K$ and strictly increasing in $w$ whenever $\epsilon(K, w + f) > \xi$, we conclude that $K^{**} > K^*$ whenever $\epsilon(K, w^* + f) > \xi$, as claimed in Corollary 2.

Proof of Corollary 3.

Under the assumptions of Proposition 2, the expected net-income per unit of capacity in equilibrium is given by:

$$\pi = LPC - w - f - d_t \cdot v - \alpha \cdot I_t$$
where $I_t$ denotes taxable income in period $t$

$$I_t = LPC - w - f - d^0_t \cdot v.$$ 

Direct substitution then yields:.................................. □

**Proof of Proposition 3.** □

**Proof of Proposition 4.**

Given any capacity investment $k$ at date 0, the firm will choose the optimal quantity in period $t$ as

$$q^*(w, \epsilon_t, k) = \begin{cases} 
K & \text{if } \epsilon_t \geq \epsilon(k, w_t) \\
q(w_t, \epsilon_t) & \text{if } \epsilon_t \leq \epsilon(k, w_t),
\end{cases}$$

where $q(w, \epsilon)$ is uniquely determined as:

$$q(w, \epsilon) = \arg\max_{\tilde{q} \leq k} \{\epsilon \cdot R^o(\tilde{q}) - w \cdot \tilde{q}\} \tag{34}$$

and

$$\epsilon(k, w) = \begin{cases} 
\bar{\epsilon} & \text{if } \bar{\epsilon} \cdot MR^o(k) \leq w \\
\frac{w}{MR^o(k)} & \text{if } \bar{\epsilon} \cdot MR^o(k) \geq w \geq \epsilon \cdot MR^o(k) \\
\epsilon & \text{if } \epsilon \cdot MR^o(k) \geq w
\end{cases}$$

We note that $\epsilon(k, w)$ is increasing in $k$ and that $q(w, k) < k$ whenever $\epsilon < \epsilon(k, w)$.

For any capacity level $k$, the present value of expected cash flows becomes:

$$\Gamma(k) = \sum_{t=1}^{T} \left\{ E[\bar{\epsilon}_t \cdot R^o(q^*(w, \bar{\epsilon}_t, k) - w \cdot q^*(w, \bar{\epsilon}_t, k)] - f \cdot k - \alpha \cdot \bar{I}_t \} \cdot \gamma^t - v \cdot k, \right\}$$

where

$$\bar{I}_t = \bar{\epsilon}_t \cdot R^o(q^*(\cdot)) - w \cdot q^*(\cdot) - f \cdot k - \hat{d}_t \cdot v \cdot k.$$ 

Define

$$\hat{\Gamma}(k) = \frac{\Gamma(k)}{(1 - \alpha) \cdot \sum_{t=1}^{T} \gamma^t}. \tag{35}$$

Following the same algebraic steps as in the proof of Proposition 2 and recalling the definition of the unit capacity cost, $c$, and the tax factor, $\Delta$, we obtain:
\[
\hat{\Gamma}(k) = E[\bar{\epsilon} \cdot R^o(q^*(w, \bar{\epsilon}, k) - w \cdot q^*(w, \bar{\epsilon}, k)] - (f + c \cdot \Delta) \cdot k
\]  
(36)

Thus,
\[
\hat{\Gamma}'(k) = \int_\epsilon^\bar{\epsilon} [\epsilon \cdot MR^o(k) - w] h(\epsilon) \, d\epsilon - (f + c \cdot \Gamma),
\]  
(37)

where \( MR^o(k) = \frac{d}{dk} R^o(\cdot) \). Since for any random variable \( X, E[f(X)] < E[max\{0, f(X)\}] \), it follows that
\[
\hat{\Gamma}'(k) \leq MR^o(k) - w - (f + c \cdot \Delta) \quad (38)
\]

By definition \( MR^o(k^o) \equiv LPC \). Therefore \( k^s \geq k^o \).

We next demonstrate that \( k^s = k^o \) if and only if the condition
\[
\text{Prob}\left[ \epsilon_t \geq \frac{w}{LPC} \right] = 1
\]  
(39)

holds. This condition is equivalent to \( \epsilon(k^o, w) = \epsilon, \)

because \( \epsilon(k^s, w) > \epsilon \) if and only if \( \epsilon \cdot MR^o(k^o) < w \). Therefore
\[
\hat{\Gamma}'(k^o) \equiv MR^o(k^o) - LPC = 0
\]

if and only if condition (39) is met.

To demonstrate the upper bound on \( k^s \) claimed in Proposition 4, we note that
\[
\hat{\Gamma}'(k) \leq [MR^o(k) \cdot \bar{\epsilon} - w] \cdot [1 - H(\epsilon(k, w))] - (f + c \cdot \Delta).
\]
\[
\leq \bar{\epsilon} \cdot MR^o(k) - LPC.
\]

Therefore \( \hat{\Gamma}'(k) < 0 \) for any \( k > k^+ \) with \( k^+ \) defined by the requirement \( MR^o(k^+) = \frac{LPC}{\epsilon} \).

Proof of Proposition 5. ■

Proof of Proposition ??.

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References


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