Optimal Relative Performance Evaluation

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Abstract

The theoretical prediction of a negative coefficient on positively correlated peer performance that underlies much of the empirical literature on relative performance evaluation, is commonly obtained from the special case where variance-covariance matrix of the performance measures is exogenously restricted to be independent of the evaluatee’s action. Using the dynamic approach of Holmström and Milgrom (1987), I study the properties of contracts that optimally condition an agent’s compensation both on his own performance and on how well he fares relative to a peer (group), when this restriction is not imposed. I show that the variance-covariance matrix is independent of the evaluatee’s action if and only if the covariance is zero and relative performance evaluation therefore is not optimal. If the covariance is non-zero, I show that the optimal contract is linear in own performance and its correlation with peer performance while, in line with the preponderance of the empirical evidence, the expected coefficient on peer performance is exactly zero.

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1 Introduction

I study optimal relative performance evaluation using the dynamic approach of Holmström and Milgrom (1987). The main distinguishing feature of my model is that normality as well as the (equilibrium) covariance of the relevant performance measures are derived rather than imposed. Several key insights are obtained. First, what constitutes a manager’s peers for the propose of performance evaluation is not simply determined by the sign of the (equilibrium) peer covariance with own performance. Instead, groups of peers are optimally formed based on the marginal effect of the agent’s second-best action on the correlation of his own performance with that of his peer(s). Second, for any random groups of peers, an agent’s compensation will be independent of (aggregate) peer performance. Peer performance will instead enter an agent’s optimal contract through realized correlation between own and peer performance. For a group of peers where an increase in the agent’s (second best) action increases the covariation with the performance of his peers, the agent’s compensation increases linearly in the coefficient of correlation. For a group of peers where increased effort leads to a decrease in covariance the effect of the correlation between own and peer performance, the effect is the opposite.

Relative performance evaluation permeates almost all aspects of human endeavour. In sports, in personal life, sports, politics, sports and in business it is more or less a truism that "achieving" and/or "performing" is at least partially judged in relative terms. Winning the Indy 500, keeping up with the Joneses, being the first man on the moon, beating an earnings forecast, losing a presidential debate or, in general, outperforming your competitors on some desirable dimension are purely relative in nature and key to how we assign credit. Surely, lap-times matters in racing, doing well financially is of value even if the Joneses are doing slightly better, playing golf on the moon is cool if it is not a first and delivering solid profits is desirable even if other managers win the profitability contest. It is hard to dispute, however, that when humans evaluate the performance others, in particular those who are stewards of our well being (such as the managers of companies in which we are stakeholders),
comparisons virtually always play a key part.

Nearly three decades have passed since the publication of the pioneering work of Antle and Smith (1986) opened the empirical inquiry into the prevalence of relative performance evaluation for managers of public corporations. Based on Holmström (1979, 1982) they proposed and tested several versions of what was perceived to be the central agency theory prediction with respect to relative performance evaluation: economy wide shocks are optimally (partially) removed from the performance related compensation of firm managers via a negative coefficient on the performance of their peers. Despite the thoroughness of their study, Antle and Smith (1986) found only what can be characterized as quite modest indications that some form of relative performance evaluation might have been at work for some part of their sample. This disappointing message, in turn, was the seed that gave rise to the relative performance evaluation puzzle that persists in the literature to this day.

The relative performance evaluation puzzle can perhaps best be described as the lack of cooperation by almost any set of data following Antle and Smith’s (1986) with the prediction of firms taking out common shocks from individual performance by deducting out some fraction of market or peer performance. The robustness of Antle and Smith’s (1986) discouraging empirical results has persisted despite the (loosely speaking) countless studies aimed at refining both the theoretical guidance and the empirical approaches used to test particular versions of this prediction. Because of the vastness of the relative performance evaluation literature, I’ll refrain from attempting to do all the idiosyncratic contributions justice here. In the spirit of this particular literature, however, what I will do instead is try to focus on the role and implications of the (in my mind) most common component of the many studies in this literature: the model(s) and their key prediction of taking out the common noise or risk component of individual performance by deducting (positively correlated) peer performance somehow (appropriately) scaled from own performance, and basing the compensation on the resulting net.

While the theoretical references used to motivate the specific empirical relative perfor-
mance evaluation \((RPE\) hereafter) hypotheses/tests have evolved over time, the starting points for this evolution, I would argue, are Holmström (1979) and, in particular, Holmström (1982). These papers develop versions of the so-called "sufficient statistic condition" and show that measures other than measures of an agent’s own performance should be used if and only if the condition is violated. The last two results in the second of these papers go a bit further. First, Theorem 8 demonstrates for two types of normally distributed production functions, that an aggregate measure of peer performance such as a market index obtained as a weighted average of the peers’ individual performances will capture all the relevant information needed for the optimal contract. Holmström (1982) is careful to point out, however, that sufficiency of a weighted average index is not the general case. Moreover, that even when a weighted average index \textit{will} suffice, it does not imply that the optimal contract generally is going to be based simply on a linear aggregate of own performance and this index. In Theorem 9, however, he proceeds to show that if there is a large number of firms exposed to the same common uncertain component, then one can approximate the solution obtained if the common uncertainty component was entirely absent.

Interesting, then, the case provided by Holmström (1982) where the optimal contract \textit{can} be based exclusively on a linear aggregate of own and a weighted average index of peer performance is one where the aggregation weight on the peer performance index is exactly minus one. Of course, the set of standard predictions in the ensuing literature does not include a negative weight of "one" on peer performance. Rather, predictions tend to come either in the “weak form” that simply predicts that \textit{some} negative weight be assigned to peer performance, or in the “strong form,” which provides predictions on both the sign and the determinants of the magnitude of the weight assigned to peer performance.\footnote{See Janakiraman et al. (1992) for a fuller discussion. Table 1 in Albuquerque (2009) provides a nice overview of the empirical relevance of these predictions. Notice the absence of a column for a prediction of "minus one."} The specific strong-form predictions are typically obtained from the standard properties of the normal distribution, either relying on Banker and Datar (1989) or (more recently) the so-called
LEN model, which is motivated by the work of Holmström and Milgrom (1987, 1991). The common component of these studies is that removing the common noise component in the optimal contract implies deducting from an agent’s own performance a positive fraction of peer performance proportional to the covariance between own and peer performance.

The lure of the normal distribution is its mathematical simplicity combined with sometimes being a reasonably descriptive way to represent standard managerial performance metrics such as stock returns. Reliance on the normal distribution is not without significant drawbacks though. Avoiding that the problem disintegrate into triviality (Mirrlees non-existence), as standard agency models based on normals normally do, requires that additional constraints be imposed. When the strong form predictions are obtained with reference to Banker and Datar (1989), the standard approach is to argue that the distribution is not quite normal after all, but somehow truncated just enough to rule out superiority of penalty contracts over those obtained using the first-order approach. In addition, to maintain the ability to aggregate own and peer performance linearly, attention must also be confined to the case where the covariance between own and peer performance is completely exogenous and independent of managerial actions. When the LEN model is used, the approach is of course to abandon optimal contracts altogether and simply impose linearity exogenously.

What is important to keep in mind, however, is that absent exogenous restrictions on the contract in models with the normal/independence specification of the production function discussed above, approximate first-best is always obtainable in which case \( RPE \) is of no meaningful economic relevance. In other words, the so-called strong form prediction that has been the predominant prediction guiding the empirical \( RPE \) inquiry does not arise from the basic underlying economic model itself; instead, is entirely accounted for by the arbitrary restrictions placed on this model. Further, while restricting attention to effort-independent noise terms is mathematical expedient and key to the linear aggregation result

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2 See for example Albuquerque (2011) and Dikolli et. al. (2011b).
3 See Mirrlees (1974) and Holmström (1982), Section 2.
4 Effort independence of the variance-covariance matrix is of course a standard LEN assumption also.
(unless, of course, if linearity is just exogenously imposed) it is arguably not particularly appealing from an economic vantage point either. Besides from being a “measure zero” type of restriction from a technical perspective, such independence seems highly implausible from an economic perspective as well. Indeed I would suggest that in most situations where peer performance is relevant, an individual’s actions have a direct impact on the degree of exposure to uncontrollable events that also are facing his peer(s).

Consider for example seemingly close peers such as McDonald’s and Burger King. By locating on opposite street corners of the same intersection, next to the same malls or freeway ramps, clearly increases their exposure to factors driving local demand. Conversely, by locating in suburban area malls away from the inner cities where many Sears stores traditionally were to be found, a retailer such as Target reduces its exposure some of the demographic-specific uncontrollable events Sears is facing. Similarly in many non-business situations that involves competing peers. Take Tour de France for example. A cyclist working hard to constantly keep within close distance to his main peers will be exposed to the exact same weather and congestion conditions as the peer at the same points of the route, while others that don’t, won’t. People that choose chicken will generally be exposed to food borne illnesses at different times than those who chose beef instead, etc. etc. The list of examples is endless.

Somewhat related, using a single shot, standard Principal-Agent model as the theoretical vehicle to study $RPE$, seems at a minimum somewhat less appropriate than it is for many other issues. Consider again the nature of the examples above. When the cyclist in Tour de France competes he is provided very frequent, if not continuous, feedback about own and peer performance, and can obviously choose to alter his actions in response. McDonald’s and Burger King can monitor one another’s pricing, product mix, promotional actions, store openings etc. and casual empiricism certainly suggests that they do follow each others’ actions closely. Moreover, if indeed assumed normality is to be at least partially justified based on the behavior of stock returns, which is the central focus of the empirical $RPE$
literature, it is clear that own and peer stock-returns provide evaluatees with more or less continuous feedback on which they can condition their actions. Frequent feedback seems almost part and parcel to many real world situations where RPE is relevant, and results obtained from a single shot model likely fail to account for the implications hereof.

Finally, the special case where linear aggregation is optimal arguably is the case where relative performance is the least relevant aspect of peer performance. Sure, the standard RPE models predict that there will be an adjustment to own performance for peer performance, but the magnitude of this adjustment is exclusively tied to absolute peer performance and does not depend on how own performance measures up relative to peer performance. That feature is of course specific to the special case where linear aggregates are sufficient. Conversely, when peer performance optimally matters but when linear aggregation will not suffice, how a given level of peer performance influences the optimal contract will necessarily depend on who outperformed who, i.e., their relative performance. Because it is only when linear aggregates won’t suffice that relative performance plays a role in calibrating the role of absolute peer performance, it would make sense to consider predictions about the nature of optimal RPE that obtains in such cases.

The alternative approach I take in this paper is intended to address these issues related to the standard approach, while maintaining the desirable features of normality. At first, my results may seem somewhat counter-intuitive as they are in sharp contrast to the conventional insights produced by the standard model(s). A key reason why my approach generates fundamentally different predictions about optimal RPE is, as pointed out by Banker and Datar (1989), in the case of normally distributed performance measures aggregate performance is sufficient only if the covariance between own and peer performance is independent of the agent’s action. When, however, one develops the normal distributions directly using the dynamic approach of Holmström and Milgrom (1987) rather than simply assuming

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5 Other papers in the theoretical literature that question the general validity of the strong form prediction for entirely different reasons than those advanced in this paper include Dye (1992), Hemmer (2004) and Celentani and Loveira (2006).
performance to be normal with the specific properties that allow for linear aggregation, it is evident that the covariance is independent of effort only in highly specific cases. In general, the covariance (at the second best level of effort) is either strictly increasing or strictly decreasing in the agent’s effort.

Consider first the case where the covariance is increasing in the agent’s effort. In this case, while, on average, higher own performance is a good thing, mismatched performance is actually a bad thing! For very low levels of peer performance, the penalty for mismatched performance therefore implies that the agent’s compensation is decreasing in own performance, while for very high levels of peer performance, the agent’s compensation is increasing (more so) in own performance. The opposite happens in the case where an increase in effort from the second best level decreases the covariance. As I show, for the continuous/normal version of the Holmström and Milgrom set-up, the implication is that the optimal contract is linearly increasing in own performance and for peers where effort is increasing (decreasing) the covariance, linearly increasing (decreasing) in the realized correlation between own and peer performance. The expected OLS coefficient on peer performance in a standard “RPE regression” is, however, shown to be exactly zero either way!

The remainder of this paper is dedicated to introducing the model, deriving the core properties of RPE under broad conditions where a simple aggregate performance measure won’t suffice but where knowing relative performance in addition to own performance actually matters. I’ll then derive the empirical predictions of this model and show that while they differ fundamentally from those obtained in the special case where aggregates are sufficient, they match the bulk of the empirical evidence accumulated to date perfectly. I will go on to discuss the economic intuition of this model’s predictions relative to the intuition of those models common in the literature and end with a few concluding remarks.
2 Model

This section develops a model where i) the focal agent’s own performance and peer performance are distributed joint normal, ii) performance feedback is frequent and iii) the effect of the agent’s action on the variance-covariance matrix is allowed to arise endogenously. My approach is based on that developed in the seminal paper by Holmström and Milgrom (1987). Accordingly, rather than assuming that the focal agent acts only once at the start of the period in question and that nothing happens until at the very end where performance then appears, effort is here assumed to be supplied, and performance is assumed to evolve continuously, over the period covered by the model.

As in Holmström and Milgrom (1987), I start by sub-dividing the fixed-length contracting horizon into \( m \) identical sub-periods where \( m \) is a positive integer. Normalizing the length of the contracting horizon to “one,” the length of each of the \( m \) sub-periods I then denote by

\[
\theta = 1/m.
\]

In each sub-period of length \( \theta \), I assume performance follows a multinomial distribution. Specifically, to make sure that the agent’s own performance becomes distributed normal in the limit as \( m \to \infty \), I restrict attention to a case where the measure of the focal agent’s performance, \( \omega_\theta \), in any sub-period of length \( \theta \) can be either positive or negative so that:

\[
\omega_\theta = \omega^i_\theta, \quad i = +, -.
\]

In each sub-period the agent influences his own performance through his action choice,

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6 Using their approach to develop the predictions also allow me to make the point as starkly as possible, that the predictions one would derive using the LEN model are fundamentally different from, and in inherent conflict with those obtain here.

7 Of course, the particular length of the period and the confinement of attention to just one period is completely inconsequential for the results using this model set-up.

8 Notice that with more outcomes per sub-period, it is not the agent’s performance that becomes distributed normal in the limit. Rather it is the Holmstrom and Milgrom (1987)- type account balances that are distributed normal which is something entirely different. I will return to the accounts in the next section.
which is here taken to be the choice of the probability of a positive outcome. Similar to Hellwig and Schmidt (2002), I use wlog the following normalization:

\[ p^* \omega^*_\theta + (1 - p^*) \omega^-_\theta = 0, \]

where \( p^* \) is the optimal second-best action for each of the \( m \) sub-periods.\(^9\) That is, in equilibrium, the expected value of (the noisy component of) own performance is normalized to zero. The history of realized own performance from the first to the last sub-period is denoted \( \overrightarrow{\pi} \), while aggregate realized own performance for the entire \( m \) sub-periods is denoted \( \Omega \).

For the correlated peer performance measure, I assume that it evolves in the same fashion as own performance, but independently of the actions of the focal agent. Thus, for each sub-period of length \( \theta \) let peer performance also be either positive or negative:

\[ \pi^+_\theta > 0 > \pi^-_\theta \]

As in the case of own performance the expected value of (the noisy component of) peer performance is normalized to zero so that

\[ q \pi^+_\theta + (1 - q) \pi^-_\theta = 0, \]

where \( q \) is the probability that peer performance in any of the sub-periods is \( \pi^+_\theta \) which, again, does not depend on the focal agent’s actions.\(^10\) The history of realized peer performance from the first to the last sub-period is denoted \( \overrightarrow{\pi} \), while aggregate realized peer performance for the entire \( m \) sub-periods is denoted \( \Pi \).

Of course, in any set-up like this, the use of peer performance in the evaluation of an agent is optimal only if the sufficient statistic condition is violated which implies that the covariance between peer performance and own performance, here denoted \( \sigma_{\omega \pi} \), is non-zero. In the spirit of the literature on RPE, the covariance is supposed to be the result of firms’ exposure to

\(^9\)\( p \in [0, 1] \) here represents all the potential actions available to the focal agent.
\(^10\)The normalizations are done to de-emphasize any implications of the relative magnitudes of \( p^* \) and \( q \) for the relative performance of the firm and the peer(s).
a “common risk” component such as economy wide conditions: when the economy is good all firms are more likely to do well while when general conditions are poor, firms are more likely to perform poorly.

From a technical vantage point, such a common component implies a non-trivial conditional probability structure between own and peer performance. Even in the very simple (binary) set-up introduced above, however, exactly how to structure this probabilistic dependence is not entirely obvious. At one extreme one could let $\eta \in [0, 1]$ denote the conditional probability $pr(\omega^i|\pi^i)$ so that a $\eta$ of 1 then represents the case where the random component of performance is always matched where as $\eta = 0$ represents the case where the random component is always mismatched. Of course, $\eta = .5$, then, represents the case of independence and thus $\sigma_{\omega\pi} = 0$.

With this particular set-up

$$p = q\eta + (1 - q)(1 - \eta)$$

or, equivalently

$$q = \frac{p + \eta - 1}{2\eta - 1},$$

where, again, $q$ is exogenous to the focal firm and, thus, this model, while $p$ is chosen endogenously by the manager of the focal firm.

Alternatively, to introduce non-zero covariance into the model one could go to the other extreme and rely on the variable $\gamma \in [0, 1]$ to denote the conditional probability $pr(\pi^i|\omega^i)$. A $\gamma$ of 1 then represents the case where the random component of performance is always matched whereas $\gamma = 0$ represents the case where the random component is always mismatched. Of course, $\gamma = .5$, then, represents the case of independence. Then,

$$q = p\gamma + (1 - p)(1 - \gamma)$$
or, equivalently
\[ p = \frac{q + \gamma - 1}{2\gamma - 1}. \]

Logically, there is no way to delineate between these two approaches as ways to introduce a common risk component here. Of course, then, any convex combination
\[ p = z\left(\frac{q + \gamma - 1}{2\gamma - 1}\right) + (1 - z)\left(q\eta + (1 - q)(1 - \eta)\right), \]
where \( z \in [0, 1] \), and \( \gamma = \eta \) is logically valid too and assuming some interior \( z \) probably would be reasonable. For simplicity and wlog I will rely on the second approach (corresponding to \( z = 1 \)) here, however.\(^{11}\)

I will also utilize the convenient (but at first glance perhaps somewhat stark) assumption that the evolution of peer performance, just like own performance, is observed as it happens by the two parties relevant to my analysis. In the case of just one sub-period, it is obviously non-controversial. In case of, for example, within-firm RPE, sporting events (again, think Tour de France or Indy 500) and arguably the key between-firm case (think stock-prices/returns), it may not just be a reasonable but also an accurate and desirable representation in general.\(^{12}\) In cases where peer performance is obtained from periodic measures compiled by units exogenous to the firm (say quarterly earnings) and where there are many sub-periods, this assumption is obviously more circumspect. However, after utilizing the simplicity and tractability it brings to its fullest, I will return later to demonstrate its benign nature in terms of its impact on the overall conclusions of this analysis.\(^{13}\)

Finally, for simplicity and also wlog, I assume the principal to be risk-neutral and normalize the agent’s coefficient of absolute risk-aversion to one so that the agent’s utility function

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\(^{11}\)Based on the analysis in the next section, it is fairly straightforward to verify that all results and conclusions are valid for any \( z > 0 \).

\(^{12}\)Core et al. (2003) suggest that stock prices account for by far the largest share of the variation in executive compensation. Accounting measures appear to have only a minimal role. Also, as can be gleaned from, for example, Table 1 in Dikolli et al. (2011a), empirical relative performance evaluation studies are based on stock prices.

\(^{13}\)Much later - like not in this paper.
takes the form

\[ u(S, c(p)) = -e^{-\left[S-\sum_{t=1}^{m} \theta c(p_t)\right]}, \]

where \( \theta c(p_t) \) is the agent’s personal cost of implementing \( p \) in one of the \( t^{th} \) of the \( m \) sub-periods.\(^{14} \) \( c' \) is used to represent the (positive) first derivative of \( c(\cdot) \) and \( S \) represents the aggregate payment under the optimal contract. The payments associated with the possible performance measure realizations in each sub-period are denoted by \( s^{ij} \), \( i, j \in \{-, +\} \), where index \( i \) is associated with the realization of own performance and index \( j \) the realization of peer performance.

### 3 Optimal Relative Performance Evaluation

Before getting started on characterizing the optimal \( RPE \) contract, I’ll provide a technical result that is key to understanding why the dynamic Holmström and Milgrom (1987) approach that I rely on here generates results fundamentally different from the standard strong form prediction discussed above.

**Lemma 1** For the dynamic model specified in the previous section, \( \sigma_{\omega} \) is independent of the focal agent’s choice, \( p \), if and only if \( \sigma_{\omega} = 0 \).

**Proof of Lemma 1.** Given that I focus on a setting where the agent can only impact the distribution of own performance through his choice of \( p \) in each of the \( m \) sub-periods and \( \gamma \) satisfies:

\[ p\gamma + (1-p)(1-\gamma) = q, \]

it is straightforward to verify that \( \gamma \) is independent of \( p \) if and only if \( p \) (and thus \( q \)) is equal to \(.5\). For \( p = .5 \), \( \sigma_{\omega} \) is easily calculated to be \( 0 \), while for \( p \neq .5 \), \( \sigma_{\omega} \neq 0 \) and is monotonic increasing in \( \gamma \). Then noting that for \( p \neq .5 \),

\[ \gamma = \frac{q + p - 1}{2p - 1}, \]

\(^{14}\)Technically, the agent’s choice can differ across the sub-periods, but since by Holmström and Milgrom (1987) Theorem 5, in equilibrium \( p^* \) is constant over time, the simpler cost representation used here will suffice.
so that

\[
\frac{d\gamma}{dp} = \frac{1 - 2q}{(1 - 2p)^2}
\]

completes the proof. ■

The key take-away from Lemma 1 is this: when normally distributed own and peer performance arise from the dynamic Holmström and Milgrom (1987) model as I use here, having both the covariance between own and peer performance be positive (or non-zero) and having this covariance be independent of the focal agent’s actions is logically inconsistent. That very assumption-combination is of course at the heart of the standard strong form prediction, whether it originates from appeals to Banker and Datar (1989) or from the LEN framework. Since a necessary condition for \( RPE \) to be optimal is that \( \sigma_{\omega \pi} \neq 0 \), in my analysis I only consider parameter values for \( p^* \) and \( \gamma \) different from .5 in which case \( \sigma_{\omega \pi} \) is non-zero, but therefore also not independent of the agent’s choice of actions.

Now, to identify the optimal properties of the incremental payments under the optimal contract generated by the realizations of the two available performance measures in any of the \( m \) sub-periods, for simplicity consider the strategies of the peers as "fixed" in the sense that each peer’s strategy is pre-determined as part of a Nash Equilibrium.\(^{15}\) The principal wishes to implement \( p^* \) for (each of the sub-periods) and his contract design problem can

\(^{15}\text{This follows Holmstrom (1982). See discussion on page....} \)
therefore be described as:

\[
\begin{align*}
\min_{s^{ij}} & \quad p^*\gamma s^{++} + p^* (1 - \gamma) s^{+-} + (1 - p^*) \gamma s^{-+} + (1 - p^*) (1 - \gamma) s^{--} \\
\tilde{\lambda}_\theta \{ & \quad p^* \gamma u(s^{++}) + p^* (1 - \gamma) u(s^{+-}) + (1 - p^*) \gamma u(s^{-+}) + (1 - p^*) (1 - \gamma) u(s^{--}) \} \\
+ & \quad \mu_\theta \{ \theta c^* p^* \gamma u(s^{++}) + \theta c^* p^* (1 - \gamma) u(s^{+-}) \\
+ & \quad \theta c^* (1 - p^*) \gamma u(s^{-+}) + \theta c^* (1 - p^*) (1 - \gamma) u(s^{--}) \\
+ & \quad u(s^{++}) \frac{d(p\gamma)}{dp} \bigg|_{p=p^*} + u(s^{+-}) \frac{d(p(1 - \gamma))}{dp} \bigg|_{p=p^*} \\
+ & \quad u(s^{-+}) \frac{d((1 - p)\gamma)}{dp} \bigg|_{p=p^*} + u(s^{--}) \frac{d((1 - p)(1 - \gamma))}{dp} \bigg|_{p=p^*} \}.
\end{align*}
\]

The corresponding optimal contract is summarized by the following lemma.

**Lemma 2** *The contract that solves the above program satisfies*

\[
\begin{align*}
\frac{1}{-u(s^{++}_\theta)} &= \tilde{\lambda}_\theta + \mu_\theta \left\{ \theta c^* + \frac{d(p\gamma)}{dp} \bigg|_{p=p^*} \right\} \equiv \lambda_\theta + \mu_\theta L^{++}, \quad (1) \\
\frac{1}{-u(s^{+-}_\theta)} &= \tilde{\lambda}_\theta + \mu_\theta \left\{ \theta c^* + \frac{d(p(1 - \gamma))}{dp} \bigg|_{p=p^*} \right\} \equiv \lambda_\theta + \mu_\theta L^{+-}, \quad (2) \\
\frac{1}{-u(s^{-+}_\theta)} &= \tilde{\lambda}_\theta + \mu_\theta \left\{ \theta c^* + \frac{d((1 - p)\gamma)}{dp} \bigg|_{p=p^*} \right\} \equiv \lambda_\theta + \mu_\theta L^{-+}, \quad (3) \\
\frac{1}{-u(s^{--}_\theta)} &= \tilde{\lambda}_\theta + \mu_\theta \left\{ \theta c^* + \frac{d((1 - p)(1 - \gamma))}{dp} \bigg|_{p=p^*} \right\} \equiv \lambda_\theta + \mu_\theta L^{--}. \quad (4)
\end{align*}
\]

**Proof of Lemma 2.** These are standard first-order conditions which here easily can be verified to be both necessary and sufficient. ■

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\footnote{By Theorem 5 in Holmström and Milgrom (1987), the optimal contract and the agent’s resulting choice can be found by solving the problem for any one of the \( m \) sub-periods.}
Corollary 1

\[ i) \quad L^{++} - L^{--} = L^{+-} - L^{-+}, \]
\[ ii) \quad L^{++} - L^{+-} = L^{--} - L^{-+}. \]

Based on Lemma 2 it is also straight-forward, using the (single) IR-constraint and one of the \( m \) identical IC-constraints, to verify that both \( \lambda_\theta \) and \( \mu_\theta \) are decreasing in \( m \) and approach zero from above as \( m \to \infty \).

Before proceeding it is worth noting that there is nothing surprising or novel about the solution to the simple one-period P-A problem as identified in Lemma 2. What is simply reaffirmed here is the standard insight, that when a positively (negatively) correlated performance measure is available and the Sufficient Statistic Condition thus does not hold for the agent’s own performance, the optimal use of this correlated performance measure is to condition the agent’s pay on matching realizations. Clearly, this role is conceptually different from the common suggestion that the optimal use of correlated performance measure is to “take out (some of) the common risk.” It does not, of course, rule out that in the limiting case where there are infinitely many such identical subsequent periods or instants and where the aggregate performance measures thus becomes normal, that the optimal contract is sensitive to peer performance and that the sensitivity is inversely related to the covariance of the two performance measures.

To gain insights into this issue consider now first introducing the "\( N - 1 \) accounts" as in Holmström and Milgrom (1987), here corresponding to each of the three different \("s^{ij} - s^{--} \neq 0"\) events that can take place in each of the \( m \) sub-periods. That is an account for the event \( ij = ++ \), an account for the event \( ij = +- \) and one for \( ij = -+ \). In each sub-period, then, credit the “++ account” with \( \theta \) if event ++ occurs and with zero if that event does not occur and similarly for the two other accounts. Let \( A^{++}, A^{+-} \) and \( A^{-+} \) denote the respective account balances of these three accounts at the end of the contracting horizon.
Then:

**Lemma 3** The agent’s optimal RPE contract can be written as

\[
S (\omega, \theta) = m \times (s_\theta^- + A^{-} \times [s_\theta^+ - s_\theta^-] + A^{--} \times [s_\theta^{+-} - s_\theta^-] + A^{+-} \times [s_\theta^{--} - s_\theta^-]).
\] (5)

The contract presented in the above Lemma simply summarizes the key insight from Holmström and Milgrom (1987) that the optimal contract for a \(m\) sub-periods problem can be obtained by solving the contract for one of the sub-periods and then simply multiply each of these 1 sub-period payments by the number of times each uniquely compensated event occurs. Again, the contract as here presented reveals little if anything about the role of peer performance and the determinants hereof. The following Theorem, however, provides the basis for gaining such insights:

**Theorem 1** Let \(m \to \infty\) so the accounts become continuous on \([0, 1]\). The optimal contract then takes the form

\[
\tilde{S} (\omega, \theta) = \tilde{\lambda}_\theta + \tilde{\beta}_\theta \{ (1 - A^{-} - A^{+-} - A^{--}) \times L^{--} + A^{-} \times L^{+-} + A^{+-} \times L^{--} + A^{++} \times L^{++} \},
\] (6)

where \(\tilde{\rho}_\theta > 0\) and \((1 - A^{-} - A^{+-} - A^{--})\) is the aggregate account balance for the "missing" \(N^{th}\) account, \(A^{--}\).

**Proof of Theorem 1.** From the derivations underlying Lemma 3, the change in the agent’s compensation associated with a change in \(A^{ij}\) is simply

\[
\ln \left( \tilde{\lambda}_\theta + \tilde{\beta}_\theta \left( L^{ij} - L^{--} \right) \right)
\]

Since in the limit \(s^{ij} - s^{--}\) becomes infinitesimal small as \(m\) grows large, the above expression converges to

\[
s^{--} + \frac{\tilde{\beta}_\theta \left( L^{ij} - L^{--} \right)}{\tilde{\lambda}_\theta + \tilde{\rho}_\theta L^{--}},
\]

where it follows from Theorem 1 that both \(\tilde{\rho}_\theta\) and the denominator are strictly positive. Then, since \(s^{--}\) can always be expressed as \(\tilde{\lambda}_\theta + \frac{\tilde{\rho}_\theta L^{--}}{\tilde{\lambda}_\theta + \tilde{\rho}_\theta L^{--}}\) the result follows. ■

The basic message here is that when the information-flow about own and peer performance is frequent and timely as in the case of stock returns, the properties of the optimal
RPE contract are exclusively determined by the likelihood ratios. That the natural logs imbedded in (5) vanishes in the limit simplifies matters significantly here. Figure 1 plots of the potential values of \( \tilde{S} (\omega', \pi') \) as a function of the account balances. Both plots are done for positive (expected) covariance between own and peer performance, that is \( \gamma > .5 \). Panel A represents the case then where \( q > p^* \), while Panel B represents the case where \( q < p^* \). Panel A can easily be verified to correspond to the cases where the covariance between own and peer performance is increasing in \( p \) at \( p^* \), while Panel B corresponds to the case where it is decreasing. And while in both cases own performance is rewarded in expectation, in the case of Panel A mismatched performance is penalized while in Panel B mismatched performance is instead rewarded. What these panels reveal is thus by no means earth shattering: they simply reaffirm the basic insights about the optimal use of correlated performance measures in standard P-A models.

Theorem 1 facilitates a simple re-representation of the optimal contract in terms of two (relatively) easy to obtain independent variables. The final Theorem provides the details.

**Theorem 2** In the limiting case where \( m \to \infty \), the optimal contract given by (6) can be written on the form

\[
\tilde{S} (\omega', \pi') = \varphi_0 + \varphi_\Omega \Omega + \varphi_\rho \rho,
\]

where \( \rho \) is the realized coefficient of correlation between own and peer performance over the time interval for which the compensation is awarded, and \( \varphi_\Omega \) and \( \varphi_\rho \) are positive constants.

**Proof of Theorem 2.** Define \( A^\Omega \equiv A^{++} + A^{+} \) and \( A^\Pi \equiv A^{++} + A^{-} \) so that \( A^\Omega \) is the “account balance” for aggregate own positive performance regardless of peer performance and \( A^\Pi \) is the “account balance” for aggregate positive peer performance regardless of own performance. \( A^\Omega = 0 (= 1) \) then corresponds to the lowest (highest) possible realization of \( \Omega \) with \( \Omega \) increasing linearly in \( A^\Omega \) while \( A^\Pi = 0 (= 1) \) then corresponds to the lowest (highest) possible realization of \( \Pi \) with \( \Pi \) increasing linearly in \( A^\Pi \). Accordingly, if \( \tilde{S} (\omega', \pi') \) can be expressed as some linear function of \( \Omega \) and \( \Pi \), it can also be expressed as a linear function of \( A^\Omega \) and \( A^\Pi \). Finally, let for simplicity \( b^{ij} \), denote the total payment received by the agent if \( A^{ij} = 1 \). From (6) it is then easily verified that \( b^{++} < b^{+} < b^{-} < b^{++} \).

Before proceeding to prove the Theorem, it is straightforward first to verify that \( \tilde{S} (\omega', \pi') \) can not be written as a simple linear function of \( A^\Omega \) and \( A^\Pi \) (and thus \( \Omega \) and \( \Pi \)). To see this fix own and peer performance at some interior levels, say \( A^\Omega = 1 - A^\Pi \). For simplicity choose \( A^\Pi = A^\Pi > .5 \). Then it is easily verified then that \( A^{+} = A^{-} \in [0, 1 - A^\Pi] \),
\(A^{++} = A^\Pi - A^{+-}\) and \(A^{--} = 1 - A^\Pi - A^+-\). Now compare the compensation received by the agent for own and peer performance \(A^\Pi\) and \(A^\Pi\) when \(A^{+-} (= A^{-+})\) is either 0 or \(1 - A^\Pi\). For \(A^{+-} = A^{-+} = 0\), we have

\[
\widetilde{S}(\overline{\omega}, \overline{\pi}) = A^\Pi b^{++} + (1 - A^\Pi) b^{--},
\]

while for \(A^{+-} = A^{-+} = 1 - A^\Pi\)

\[
\widetilde{S}(\overline{\omega}, \overline{\pi}) = (2A^\Pi - 1)b^{++} + (1 - A^\Pi)b^{+-} + (1 - A^\Pi)b^{-+}.
\]

Since \(b^{+-} < b^{--} < b^{++}\) and \(A^\Pi > 2A^\Pi - 1\), the first line is strictly greater than the second. Accordingly, \(A^\Pi\) and \(A^\Pi\) do not uniquely determine \(\widetilde{S}(\overline{\omega}, \overline{\pi})\) and \(\widetilde{S}(\overline{\omega}, \overline{\pi})\) can therefore not be expressed simply as some function of \(\Omega\) and \(\Pi\) alone.

Next, fix again own performance at some level \(A^\Pi\), but instead of fixing peer performance fix instead the realized correlation coefficient of own and peer performance, \(\rho\), at some level, say \(\overline{\rho}\). Since \(A^\Omega \equiv A^{++} + A^{+-}\) and \(\rho \equiv A^{++} + A^{+-} - A^{+-} - A^{--}\), clearly there are infinitely many account balance combinations that corresponds to any (interior) \(A^\Pi, \overline{\rho}\) pair. However, for every fixed \(A^\Pi, \overline{\rho}\), using their respective definitions (and the fact that \(A^{++} + A^{+-} + A^{-+} + A^{-+} = 1\)), we can re-express \(\overline{\rho}\) as

\[
\overline{\rho} = 1 - 2A^{+-} - 2A^{-+}.
\]

Consider then continuously changing \(A^{+-}\) from the lowest to the highest value consistent with \(A^\Pi, \overline{\rho}\). From the above it is then straight forward to verify that \(\frac{dA^{++}}{dA^{+-}}\bigg|_{A^\Pi, \overline{\rho}} = \frac{dA^{+-}}{dA^{+-}}\bigg|_{A^\Pi, \overline{\rho}} = -1\). Then it follows directly from (the corollary to) Lemma 2 and Theorem 1 that \(\frac{d\widetilde{S}(\overline{\omega}, \overline{\pi})}{dA^{+-}}\bigg|_{A^\Pi, \overline{\rho}} = 0\) and thus the optimal compensation for a given level of \(\Omega\) and \(\rho\) is independent of the specific combination of the associated account balances.

Due to this property of the optimal contract it is always possible to rewrite the optimal contract as some linear function of \(A^\Omega, A^{+-}\) and \(A^{-+}\), hereafter denoted \(\psi (A^\Omega, A^{+-}, A^{-+})\). Then, for \(A^\Omega = 1, \rho = 1\) if \(A^{+-} = 0\) and \(\rho = -1\) if \(A^{+-} = 1\) so that for \(A^\Omega = 1, \rho = 2 \left((1 - A^{+-}) - \frac{1}{2}\right)\). For \(A^\Omega = 0, \rho = 1\) for \(A^{+-} = 0\) and \(\rho = -1\) for \(A^{+-} = 1\) so that for \(A^\Omega = 0, \rho = 2 \left((1 - A^{+-}) - \frac{1}{2}\right)\). Then, again due to the "linearity in accounts" the optimal compensation corresponding to a particular \(\rho\), say \(\overline{\rho}\), is given as

\[
A^\Omega \times \psi \left(1, \frac{1 - \overline{\rho}}{2}, 0\right) + (1 - A^\Omega) \times \psi \left(0, 0, \frac{1 - \overline{\rho}}{2}\right) = \psi \left(0, 0, \frac{1 - \overline{\rho}}{2}\right) + A^\Omega \left[\psi \left(1, \frac{1 - \overline{\rho}}{2}, 0\right) - \psi \left(0, 0, \frac{1 - \overline{\rho}}{2}\right)\right].
\]
Further note that
\[
\psi \left( 1, \frac{1 - \bar{p}}{2}, 0 \right) - \psi \left( 0, 0, \frac{1 - \bar{p}}{2} \right) = b^- + \left( 1 - \frac{1 - \bar{p}}{2} \right) (b^{++} - b^-) - \left( \frac{1 - \bar{p}}{2} (b^+ - b^-) \right) = b^{++} - b^-(> 0),
\]
and therefore the reward for own performance is independent of \( \rho \).

Now also note that since the optimal compensation is linear in \( \Omega \) and \( dS/d\Omega \) is independent of \( \rho \), \( dS/d\rho \) is also independent of \( \Omega \). Then, given that
\[
dS \bigg|_{A^\alpha=0} = -\frac{b^- - b^-}{2} = \frac{b^{++} - b^-}{2} = \frac{dS}{d\rho} \bigg|_{A^\alpha=1},
\]

independent of own performance, \( \Omega \). Since for any \( \{\Omega, \Pi\} \) pair, the feasible combinations of \( A^{+\alpha} \) and \( A^{+\Pi} \) correspond uniquely to a particular level of \( \rho \), this concludes the proof.

Clearly, the use of peer performance in the optimal dynamic contract derived here is more complex and conceptually different from the common suggestion in the \( RPE \) literature. That a simple linear combination of own and peer performance does not suffice to determine compensation does, of course, rule out that, say, on average the optimal contract is sensitive to peer performance and that the sensitivity is inversely related to the covariance of the two performance measures in which case the standard empirical predictions could still be valid. To address this question, Lemma 4 summarizes the implications for a standard \( RPE \) \( OLS \) regression of the properties of the optimal \( RPE \) contract derived here.

**Lemma 4** Using the optimal contract, \( \tilde{S} (\bar{\omega}, \bar{\pi}) \), as the dependent variable in the following \( OLS \)-regression on aggregate own and peer performance,
\[
\tilde{S} (\bar{\omega}, \bar{\pi}) = \tilde{\beta}_0 + \tilde{\beta}_\Omega \Omega + \tilde{\beta}_\Pi \Pi + \tilde{\varepsilon},
\]
for a large sample of identical firms, the expected regression coefficients satisfy
\[
\tilde{\beta}_\Omega > \tilde{\beta}_\Pi = 0.
\]

**Proof of Lemma 4.** The expected relative frequency of realized performance and incremental compensation in each of the \( m \) independent sub-periods are \( A^{+\alpha} \equiv (1 - A^{+\gamma} - A^{+\gamma} - A^{++}) = m (1 - p^\ast) \gamma, A^{+\alpha} = mp^\ast (1 - \gamma), A^{+\alpha} = m (1 - p^\ast) (1 - \gamma) \) and \( A^{++} = mp^\ast \gamma \), where the
first two accounts track relatively low peer performance while the last track relatively high. The OLS-regression then solves for the pair \( \bar{s}^- \) and \( \bar{s}^+ \) that minimizes

\[
m (1 - p^*) \gamma (\mu_g L^{--} - \bar{s}^-)^2 + m p^* (1 - \gamma) (\mu_g L^{+-} - \bar{s}^-)^2
\]

and

\[
m (1 - p^*) (1 - \gamma) (\mu_g L^{-+} - \bar{s}^+)^2 + m p^* \gamma (\mu_g L^{++} - \bar{s}^+)^2.
\]

Using \( \frac{d \bar{s}}{dp} = \frac{1 - 2q}{(1 - 2p)^2} \gamma \) it is easily verified that \( \bar{s}^- = \bar{s}^+ \) and accordingly that \( \beta_{\Pi} = 0 \). The proof that \( \tilde{\beta}_{\Omega} > 0 \) is mechanically identical and is thus left for the reader. ■

The intuition for Lemma 4 can be gleaned from the plots in Figures 2 and 3. Figure 2 collapses Figure 1 panel A into two two-dimensional plots, the first in the "own" dimension (Panel A) and the second in the "peer" direction (Panel B). The bold lines through the plotted areas in Panel B of both figures are the mean-squared-error-minimizing (OLS) lines for either "sample" taking into account the underlying joint density. Accordingly, the slope of these lines represent the OLS regression-coefficients one would expect to obtain from the (standard RPE) regression in the above Lemma. Figure 3 does the same for Panel B of Figure 1 as Figure 2 does for Panel A of Figure 1. The key take-away is that \( \beta_{\Pi} \) is predicted to be zero whether the covariance is increasing or decreasing in own effort. The skeptic can easily verify the same would be true if the correlation between own and peer performance had been negative. That \( \beta_{\Omega} \) is always positive here independent of the sign of the covariance is also straight-forward to verify.

4 Discussion

The results obtained in the previous section differ fundamentally from the standard predictions in the RPE literature. A key difference is clearly (but in my mind entirely unsurprisingly) that in the general case, it is not the sign of the covariance that drives optimal RPE; it is what the agent can do about the covariance that matters! From an empirical vantage point that is, at first glance, somewhat unfortunate as it moves the predictor from being an equilibrium property that can be easily estimated (covariance) to an out-of-equilibrium one.
(change in covariance) than cannot, at least not directly. If one steps back from the technical aspects of the RPE results, both the common ones in the existing aggregate performance evaluation literature and those presented here, and instead considers the general economic intuition behind them I think the ones presented here make a lot more sense; both in terms of what they say about identifying the relevant peers and in terms of the observable variables that should be able to explain observed compensation.

To see why this is, consider again the Holmström (1982) Theorem 9 case. While insightful and elegant, I don’t see much enthusiasm in the literature for the case that the predicted weight on peer performance should be minus one.\(^{17}\) The reason, I suspect, is that as discussed in the introduction it corresponds to a case where only relative performance, not own performance, matters. Generally, I think, most people’s experience with RPE is less extreme. Winning is not enough, for example. If you played well typically matters too. Doing slightly better than Enron is not likely to elicit much applause, having the fastest car on the block is not really that cool if it is a ’87 Yugo GV, having the fewest publications in the department is a bit different if fewest means 20 than if it means zero and so on.\(^{18}\) In other words, a negative weight of one doesn’t correspond to the way we intuitively think about RPE and it shouldn’t be surprising either, that it isn’t what the data suggest.

I would argue that the same type of reality check on the standard (strong form) prediction in the existing RPE literature does not actually correspond to most people’s intuition or experiences either. I cannot think of (m)any good economic examples where independent of whether the individual did better or worse than his peers, individual performance is simply adjusted (down) by some fraction of average peer performance. Alternatively, I don’t think that people exposed to RPE are ever satisfied just to be provided with some summary statistic without being able to discern how they actually performed relatively to their peers.

\(^{17}\) Allocation of fixed bonus pools or total raises available would fit as an example, but is in my mind not the kind of RPE we expect for executives that are typically the subjects of study in this literature.

\(^{18}\) Furthermore, while pundits for many years have called for, for example, indexing Executive Stock Options thus effectively placing a negative weight of one on peer performance, that idea appears to have gained very little traction as a practical matter.
either. Telling students that they got a B+ is not going to be sufficient without also telling them the mean for their peers. Or telling a faculty member that their raise is 3% is not going to work that well, unless it is accompanied with information about what the peers got and why. However, if the conditions that underlie the standard “strong form” prediction are met, providing anything beyond the final aggregate should really be of no significance other than perhaps to verify that the aggregation was done correctly. And again the data agrees with the intuition/casual empiricism here: it is just not how RPE appears to be done in general.

The key insights generated by my analysis seem, at least to me, fare a lot better in this gut-check contest. First of all, unlike in the case of Holmström’s (1982) Theorem 9 both own and peer performance matter. Second, relative performance matters too, because a simple linear aggregate as in Banker and Datar (1989) is not sufficient in general to communicate the implications of own and peer performance. More importantly perhaps, the notion that it is the change in the covariance that is key appears intuitively consistent with how we go about RPE in all kinds of standard situations. Think, for example, about a parent trying to encourage his/her child to excel in High School. How to best do this may, of course, differ by the nature of the school, culture, etc., but I don’t believe the typical approach is that the parent compares the kid with the average student in the school or deducting some fraction of average test scores from their own child’s scores as part of metering out a reward (or punishment). Rather, it is to identify peers that the student should try to be more like (think honor roll) and/or peers that the student should try and be less like (I’ll leave identification of this group to the reader’s own experience and imagination).

This approach to identify not one but multiple groups of peers seems so much more in line with how I believe we think about RPE in general. For a cell phone maker, for example, my guess is that the benchmark used for RPE is not based on a weighted average of Apple and RIM. Rather, RPE is used to encourage managers at this point in time to make their firms more like the Apples of the world and less like the RIMs. Politicians argue that we
should be less like Greece and more like whatever country they view as having desirable
properties to be emulated. People have heros and role models they aspire and strive to be
like and similarly reference groups they actively try and disassociate themselves from. The
examples are endless and I’d therefore argue that this kind of “good peer, bad peer” \textit{RPE}
suggested by my very basic analysis is the norm while the use of an “average peer” is at best
a marginal and, as the evidence suggests, clearly an empirically irrelevant exception.

Furthermore, the task of identifying the relevant peer groups I think is actually quite
straight forward. Becoming more like an ideal group implies as far as I am concerned not
simply boosting ones performance relative to this group, but more importantly increasing
ones covariance with it. Becoming less like another group of peers have the same relative
consequences but also implies reducing the covariance with that group. And as I have shown
in the previous section, reward structures that provide incentives to become more like some
group differ fundamentally from reward structures that provide incentives to distance oneself
from another group. Moreover, in neither case does the optimal reward structure disintegrate
into simply placing a positive weight on own performance and a negative weight on average
peer performance. Instead, it adjusts own performance with relative performance as captured
by the realized correlation between own and peer performance.

5 Conclusion

I develop a set of agency theory based predictions about \textit{RPE} using the basic framework
of Holmström and Milgrom (1987). As I show, that generally the covariance between jointly
normal own and peer performance is not independent of the agent’s effort in this type
of model. Accordingly, as pointed out by Banker and Datar (1989), linearly aggregated
performance measures are therefore not sufficient and a contract written on own minus
scaled peer performance is not optimal. What is optimal depends on how the agent’s effort
impacts the covariance between own and peer performance. If it increases the covariance
the optimal contract, while increasing in own performance, rewards "similar" performance with peers. If the covariance is decreasing, the optimal contract instead rewards "dissimilar" performance. As a result, the optimal contract varies in the correlation between own and peer performance but, consistent with the empirical evidence, optimal RPE is independent of aggregate peer performance.
References


6 Figures

Figure 1. $\tilde{S}(\omega, \pi)$ for $\frac{d\gamma}{dp} > 0$ (Panel A) and $\frac{d\gamma}{dp} < 0$ (Panel B).
Figure 2. Compensation from Figure 1, Panel A levels plotted against own (Panel A) and peer (Panel B) performance.
Figure 3. Compensation from Figure 1, Panel B levels plotted against own (Panel A) and peer (Panel B) performance.